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Rationality of the personal loan interest-rate markups of banks

銀行對個人貸款加碼利率合理性之研究

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Abstract: This study provides a model of banks' personal loan interest-rate markups by analyzing their expected profit maximization. Employing 804 personal loan cases from one Taiwanese bank, our empirical findings show that this model is able to identify the personal loan interest cut-off rates for each risk segment for bank profit maximization in order to determine personal loan interest-rate markups in a more accurate and rational manner. The interest-rate markups identified by this model can serve as a reference for banks' projections regarding optimal interest-rate markups for personal loans.

Keywords: Credit scoring model, credit quality, credit risk, interest-rate markups.

1. Introduction

Credit risk is the main source of banking problems (The Basel Committee on Banking Supervision, 2004), and the nature of loans implies different credit risks that affect investors' performance evaluation towards banks (Chen, 2011). Studies about the credit risk model can be divided into accounting-based models that adopt historical financial data and market-based models that use information

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on the equity and bond markets (Lin and Chang, 2009). The quality of credit risk management determines the operational performance of banks. In order to attract more customers under intense competition in the financial market, banks are likely to lower their credit-granting standards, exposing them to high default risks. Thus, reducing redundant credit risk is a critical challenge for banking operations, and appropriate credit-granting risk management can shield banks from unnecessary credit losses that reduce profits and even lead to bankruptcy (Graham and Horner, 1988; Hess, Grimes, and Holmes, 2009).

Because financial products are the essence of banks' customer products, financial risk is their main risk (Chen, Chang, and Chang, 2004). A borrower's profit efficiency is the most important determinant of the investor's perception of bank loan quality and subsequent loan defaults (Chi, 2015). To enhance credit-granting risk management, banks should employ effective evaluation tools, such as utilizing credit scoring models to reduce the control costs of credit risk (Jappelli and Pagano, 2002). Numerous scholars argue that credit scoring models are able to identify critical risk variables that help prevent the occurrence of loan defaults (Crook, Hamilton, and Thomas, 1992; Dinh and Kleimerier, 2007; Steenackers and Goovaers, 1989) Therefore, credit scoring models are a necessary tool for banks to select loan customers.

When determining whether to offer loans to borrowers, banks conduct a credit rating of the borrowers to evaluate their default risk. However, there exists an adverse selection problem in the decision of offering interest-rate makeups for customers' loans under an inaccurate evaluation. An underrating evaluation could exclude good borrowers with comparatively low risk from obtaining loans, which results in the bank losing some potential profit sources, because a high interest-rate markups can cause customers to abandon their loans or transfer to other banks after comparing the interest rates of various banks. An overrating evaluation could include bad borrowers with comparatively high risk, resulting in the bank being exposed to too much default risk, because a low interest-rate markups can attract customers to apply for loans or transfer from other banks. Therefore, setting the optimum interest-rate markups helps retain good borrowers and avoid to burden of excess default risk. Determining how to set the optimum interest-rate markups is the key mechanism or technique for the operational performance of banks.

The construction of interest-rate markups models determines the operational performance of banks, which is as vital as constructing credit models for offering loans. Stein (2005) found that accurate credit evaluation methods can increase the profits of general small- and medium-sized banks. Dinh and Kleimerier (2007) stated that reducing the default ratio from 3.3% to 2% using the credit model proves that credit models do assist banks in controlling risk. Steenackers and Goovaers (1989) identified the optimal method for recognizing the accuracy of defaults, enabling banks to apply this method to credit scoring models. Hasumi and Hirata (2010) established an evaluation model for maximum profits to calculate the optimal loan size and profit amount. However, these studies did not explore the rationality and importance of interest-rate markups. Under the premise of profit maximization, this study develops a model for interest-rate markups by determining the optimal cut-off rate in each markup interval. Among the various cut-off rates, each cut-off rate for the markup interval.

The data used for this study encompass credit-granting cases from 2009 to 2015, including loans against collateral, unsecured loans, and consumer loans, sourced from the database of one bank in Taiwan. After eliminating the observation values with insufficient variable data, 804 effective loan entries remained; of which, 456 were regular customers and 348 were defaulting customers. This study employs logistic regression analysis to examine the default risk variables. The variables that correlate positively with default are occupation, the amount of guarantors, the period of credit-granting business, the period of loans, and deposit performance. The variables that are negatively correlated with default are annual income, loan amount, the holding of credit cards, government preferential policies, and the duration of active bank accounts. Regarding the calculation of interest-rate markups, the higher the probability is for a breach of contract, the higher the risk premiums will be for the bank requests, resulting in a high interest-rate markup. Low probability can lead to low risk premiums, which leads to a low interest-rate markup. This inference has been supported by empirical results. Moreover, the empirical results herein prove this inference, that is, the optimum cut-off rate provided by the interest-rate markups model can maximize bank profits.

The remainder of the paper is as follows. Section 2 investigates related

studies and the empirical results. Section 3 describes the empirical model herein and the detailed procedures for identifying the optimum cut-off rate. Section 4 presents the research data, samples, and variables chosen for this study. Section 5 details the empirical results, and Section 6 is the conclusions.

2. Literature review

Credit evaluation strategies and decision-making are the most essential and challenging aspects of the loan application or acceptance process. Based on credit evaluation results, banks use the pricing strategies of interest rates to calculate the maximum profit potential and required risk premiums of loans, which are associated with whether customers will default and impact banks' profit. Dinh and Kleimerier (2007) maintained that in addition to controlling related costs and expenses, banks must employ credit evaluations to control the loan default rate and achieve the expected profits. Banks primarily approve loans based on credit evaluations that use the characteristics of applicants (e.g., their occupation and income) as variables to calculate the rating or score of each applicant. Banks employ differing variables and credit evaluation methods. However, in addition to external economic factors, the occurrence of defaults is related to insufficient credit checking information analysis and credit evaluation methods that do not reflect true or real situations and are not sufficiently standardized. Therefore, credit evaluations and the pricing of interest rates are both equally crucial for banks.

Credit scoring models

Beaver (1966) proposed a model for forecasting a financial crisis based on the univariate model. Altman (1968) developed a Z-score linear model using discriminant analysis to identify a formula of five financial ratio variables that are highly accurate for forecasting corporate bankruptcy. Scholars have also examined credit scoring models (Hand and Henley, 1997; Thomas, 2000), with numerous scholars having improved credit scoring models in the last 10 years (Anderson, 2007; Crook, Edelman, and Thomas, 2007; Siddiqi, 2006; Thomas, 2009; Thomas, Edelman, and Crook, 2002), which have attracted the attention of banks to credit scoring models. Both large banks and community banks have employed credit scoring models to reduce defaults (Berger, Cowan, and Frame, 2011).

A credit scoring model is a forecast model that assists financial institutions in evaluating the credit risk of customers and increasing profits from loans (Řezáč and Řezáč, 2011). Additionally, credit scoring models can identify those factors that influence defaults (Crook *et al.*, 1992; Dinh and Kleimerier, 2007; Steenackers and Goovaers, 1989). The empirical results by Dinh and Kleimerier (2007) indicate that credit scoring models can reduce the default ratio from 3.3% to 2%, thus verifying that credit scoring models are beneficial for bank risk management and control.

Steenackers and Goovaers (1989) developed an optimal method for recognizing or identifying the accuracy of defaults, enabling banks to apply this method to credit scoring models. Crook *et al.* (1992) compared the forecast results of multiple credit scoring analysis methods (e.g., stepwise regression and equal weights for all variables) and identified the analytical method that provides superior forecasts. Avery, Calem, and Canner (2004) found that a high unemployment rate can lead to a high default rate; that is, the unemployment rate is positively correlated with defaults. The occurrence of defaults is highly or completely correlated to people's situational factors. The most significant contribution of this research is the incorporation of situational factors into the credit scoring variables. Carling, Jacobson, and Roszbach (2001) contended that in addition to customer defaults causing bank risk, banks must consider the effect of prepayment to calculate profits practically and realistically and with greater accuracy.

The pricing of interest rates

Extant studies in the literature have not addressed the rationality and appropriateness of interest-rate markups. Instead, numerous studies have examined only overall profit calculation and cost control under banks' existing interest rate standards. Dinh and Kleimerier (2007) used the credit scoring model to identify the default variables and established a profit calculation equation to assist banks in saving costs and increasing earnings and profits.

Stein (2005) utilized the receiver operating characteristic (ROC) method to

examine bank pricing strategies, by considering bank risks and costs, and found that effective credit scoring analysis methods enable small- and medium-sized banks to increase their profits. When calculating bank profit, Carling *et al.* (2001) argued that in addition to default loss, the loss resulting from prepayment must also be considered to calculate profits more accurately.

3. Empirical Model

The credit scoring model was derived from the Z-score model. Altman (1968) analyzed and forecasted corporate bankruptcy using the multivariate discriminant analysis method, which was further employed for credit scoring classification. The method categorizes samples into several groups based on their characteristics and constructs discriminant functions based on the sample values. By categorizing samples using discriminant function values, we can evaluate the probability of corporate defaults and conduct credit ratings. Multivariate discriminant analysis enables scholars to consider multiple financial indicators, evaluate overall corporate performance, and identify the financial ratios with discriminant ability. Ohlson (1980) and Dinh and Kleimerier (2007) used logistic regression analysis to forecast borrower defaults.

The purpose of this study is to develop a model that can be a reference for banks. On the premise of competition, we look to find the optimal interest rate on loans so as to not impact profit when the rate is too low or drive potential customers to other competitors when the rate is too high. Therefore, this model first excludes the samples of defaulting customers, and then we conduct analysis and screening on regular customers. Next, we gradually and categorically identify the standard of risk premium under different risk levels of loans so as to obtain the optimum interest rate for a specific customer. The better (worse) the condition is for a customer, the lower (higher) the loan interest rate will be. Our goal is to develop a model to discover the most optimal interest rate given the parameters set up.

Empirical model

The model uses logistic regression to forecast a dichotomous or ordinal variable value, and the parameter estimation method is the maximum-likelihood

method. The aim of this model is to identify the linear relationship between the independent and dependent variables. For this study's model, we reference and revise the model developed by Dinh and Kleimerier (2007) as shown below.

$$Z_j = W_x = w_0 + w_1 x_{j1} + w_2 x_{j2} + \dots + w_k x_{jk},$$

where, w_0 : constant term; w_k : the coefficient of the k^{th} variable; x_{jk} : the value of applicant *j* for the *k* variable; and Z_j : the Z value of applicants, where 0 denotes regular customers and 1 denotes defaulting customers. The factors influencing credit-granting quality are used as the explanatory variables to analyze the variables that influence a default. After coding and analyzing the existing borrower information, we execute a logistic regression to calculate the π value of applicants and identify the t variables influencing default.

$$\pi_j = \frac{1}{1 + e^{-(W_X)}},$$

where π_j : the default probability of applicant *j*. This value ranges between 0 and 1; the higher the value, the higher the default probability.

After identifying the variables influencing default and eliminating possible defaulting customers, banks should set rational interest rates. Rational interest markups help retain good customers and avoid attracting bad customers for maximizing profits. In this study we develop a profit maximization model that can categorize and sum up retrievable and lost profits, identify the cut-off rate for maximum profit for each markup interval to use as the standard for determining interest rates, and determine the loan interest rate for each customer based on the π value.

We divided the markup rates into *n* intervals $(I_1, I_2, ..., I_n)$ and use the π values calculated in the above equation as the standard for determining or pricing interest rates (Table 1).



Table 1Markup intervals and interest rates

We note that C_1 to C_n represent the cut-off rates of each markup interval. If the π value of the customer is smaller than C_1 , then the I_1 interest-rate should be offered. If the π value is between C_1 and C_2 , then the I_2 interest-rate markup should be offered to the customer. Further interest-rate markups can be determined based on this principle. If the π value of the customer is larger than C_n , then the interest-rate markup I_{n+1} should be offered to the customer. To simplify the empirical model, we assume that the prime rate = 0 and the loan amount = 1. We then substitute these values into the profit calculation model provided below to identify the optimum cut-off rates, C_1 to C_n , of each markup interval.

The profit equation calculation procedures are presented below. The first procedure is to input all the loan data (including defaulting and regular customers) into the model to capture the cut-off rate C_{n+1} , so that it can distinguish defaulting customers D. However, after the second step of the model, the reason for excluding defaulting customers is so that we can input the data of regular customers into the model, in order to calculate the optimal interest-rate markups.

Procedure 1

Customers are divided into credit-granted customers $I_1 \sim I_{n+1}$ and denied customers (*D*) to calculate the optimum cut-off rate, C_{n+1} (Table 2).

	Table 2Procedure One		
		Forecast π val	lues
The actual credit value of borrowers (N = total loan amount)	$ \substack{(L) \ l_1 \sim l_{n+1} \\ (H) \ D} $	(l) $I_1 \sim I_{n+1}$ $L_l^{C_{n+1}}$ $H_l^{C_{n+1}}$	$(h) D L_h^{C_{n+1}} H_h^{C_{n+1}}$
1.Based on the Z value of each loan applica $H_l^{C_{n+1}}$, and $H_h^{C_{n+1}}$, and the total calculate $\frac{H_l^{C_{n+1}}}{N}$, and $\frac{H_h^{C_{n+1}}}{N}$, and the total probabilit 2. <i>L</i> and <i>H</i> : the actual credit value of borr (<i>L</i>) $I_1 \sim I_{n+1}$: offered the interest-rate m (<i>H</i>) <i>D</i> : listed as customers denied loans 3. <i>l</i> and <i>h</i> : forecast π values. (<i>l</i>) $I_1 \sim I_{n+1}$: when $\pi \leq C_{n+1}$, the interest (<i>h</i>) <i>D</i> : when $\pi > C_{n+1}$, borrowers are l	ed amount is N. The provided amount is N. The provided amount is N. The provided amount is $\frac{L_{l}^{C_{n+1}}}{N} + \frac{L_{h}^{C_{n+1}}}{N} + \frac{H_{l}^{C_{n+1}}}{N}$ owers based on their actual est-rate markups $I_{1} \sim I_{7}$	robability of each quad $\frac{E_{n+1}}{N} + \frac{H_n^{C_{n+1}}}{N} = 1$ d on the actual credit value. m_{n+1} are offered to borrow	Frant is $\frac{L_l^{\hat{c}_{n+1}}}{N}$, $\frac{L_h^{\hat{c}_{n+1}}}{N}$,

Profit function C_{n+1}

$$Profit = \left(R_{I_{1\sim(n+1)}} \times \frac{L_{l}^{C_{n+1}}}{N}\right) - \left[\left(1 + R_{I_{1\sim(n+1)}}\right) \times \frac{H_{l}^{C_{n+1}}}{N}\right] - \left(R_{I_{1\sim(n+1)}} \times \frac{L_{h}^{C_{n+1}}}{N}\right) + \left(1 \times \frac{H_{h}^{C_{n+1}}}{N}\right)$$
(1)

- $L_l^{C_{n+1}}$ = The number of loans when the interest rates of $I_1 \sim I_{n+1}$ based on the forecast π value are offered to borrowers and the interest rates based on the actual credit value of borrowers are $I_1 \sim I_{n+1}$.
- $H_l^{C_{n+1}}$ = The number of loans when the interest rates of $I_1 \sim I_{n+1}$ based on the forecast π value are offered to borrowers and the loan based on the actual credit value of the borrower yields a denied result.
- $L_h^{C_{n+1}}$ = The number of loans when the loan based on the forecast π value yields a denied result and interest rates based on the actual credit value of borrowers are $I_1 \sim I_{n+1}$.
- $H_h^{C_{n+1}}$ = The number of loans when the loans based on the forecast π value and the actual credit value of borrowers both yield a denied result.

 $R_{I_{1\sim(n+1)}}$ = The average interest rate of loans $I_{1\sim(n+1)}$.

The principle of this profit calculation is to sum up the accurate forecast profits and incorrect forecast losses to calculate the profit maximization cut-off rate - that is, the optimum cut-off rate: C_{n+1} .

Term 1

 $\left(R_{I_{1\sim(n+1)}} \times \frac{L_{l}^{C_{n+1}}}{N}\right)$: The profits from accurate forecasts of borrowers' risk given interest rates of $I_{1} \sim I_{n+1}$. Since the number of loans offered at each interest rate interval for $I_{1} \sim I_{n+1}$ is not obtained in *procedure 1*, the product of the average interest rate $(R_{I_{1\sim(n+1)}})$ and probability $\left(\frac{L_{l}^{C_{n+1}}}{N}\right)$ is used for the expected returns for this outcome.

Term 2

 $\left[\left(1+R_{I_{1\sim(n+1)}}\right)\times\frac{H_{l}^{C_{n+1}}}{N}\right]$: The default losses of underestimating borrowers' risk given interest rates of $I_{1}\sim I_{n+1}$. Loan applications would have been denied based on the actual credit value of borrowers; however, the results of the forecast model suggest that the bank accept their loan applications, which leads to default

losses $(1 + R_{I_{1\sim(n+1)}})$; therefore, $(1 + R_{I_{1\sim(n+1)}})$ multiplied by the probability $(\frac{H_l^{C_{n+1}}}{N})$ is the expected losses for this outcome.

Term 3

 $\left(R_{I_{1\sim(n+1)}} \times \frac{L_{h}^{C_{n+1}}}{N}\right)$: The opportunity cost caused by incorrectly denying the loan applications given interest rates of $I_{1} \sim I_{n+1}$. The bank denied the customers' loan applications based on the π value and lost profit $(I_{1\sim(n+1)})$; therefore, the average interest rate $(R_{I_{1\sim(n+1)}})$ multiplied by the probability $\left(\frac{L_{h}^{C_{n+1}}}{N}\right)$ is the expected profit losses.

Term 4

 $\left(1 \times \frac{H_h^{C_{n+1}}}{N}\right)$: The losses avoided by accurate forecasts of borrowers' risk and the rejection of these borrowers.

Procedure 2

are denied loans.

Customers are divided into markup groups $I_1 \sim I_n$ and $I_{n+1} + D$ to calculate the optimum cut-off rate: C_n (Table 3).

Table 3Procedure Two

		Foreca	st π values
		(l) $I_1 \sim I_n$	(<i>h</i>) $I_{n+1} + D$
The actual credit value of borrowers	(L) $I_1 \sim I_n$	$L_l^{C_n}$	(h) $I_{n+1} + D$ $L_h^{C_n}$ $H_h^{C_n}$
(N = total loan amount)	(<i>H</i>) I_{n+1} +D	$H_l^{C_n}$	$H_h^{C_n}$
1. Based on the Z value of each loan applican $H_l^{C_n}$, and $H_h^{C_n}$, and the total calculated and			
and $\frac{H_h^{C_n}}{N}$, and the total probability is			
$\left[\frac{L_{l}^{C_{n}}}{N}\right] + \left[\frac{L_{h}^{C_{n+1}}}{N} + \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N}\right] + \left[\frac{H_{l}^{C_{n+1}}}{N} + \frac{H_{l}^{C_{n}}}{N}\right]$	$\frac{-H_l^{C_{n+1}}}{N} + \left[\frac{H_h^{C_{n+1}}}{N} + \frac{H_h^{C_{n+1}}}{N}\right]$	$\frac{{}^n - H_h^{C_{n+1}}}{N} \bigg] = 1$	
2. L and H: the actual credit value of borrow	vers		
(L) $I_1 \sim I_n$: offered the interest-rate marku	ups $I_1 \sim I_n$ based on the	e actual credit val	ue of borrowers.
(H) $I_{n+1} + D$: offered the interest-rate	markups I_{n+1} or a de	nied result based	on the actual credit
value of borrowers.			
3. <i>l</i> and <i>h</i> : forecast π values.			
(<i>l</i>) $I_1 \sim I_n$: when $\pi \leq C_n$, the interest-rate	e markups $I_1 \sim I_n$ are c	offered to borrowe	ers.
(h) $I_{n+1} + D$: when $\pi > C_n$, the interest	t-rate markups I_{n+1} a	are offered to bor	rowers or customers

Profit function C_n

$$Profit = \left(R_{I_{1\sim n}} \times \frac{L_{l}^{C_{n}}}{N}\right) - \left[\left(1 + R_{I_{1\sim (n+1)}}\right) \times \frac{H_{l}^{C_{n+1}}}{N} + \left(R_{I_{n+1}} - R_{I_{1\sim n}}\right) \times \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N}\right] - \left(R_{I_{1\sim (n+1)}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N}\right) + \left(1 \times \frac{H_{h}^{C_{n+1}}}{N} + R_{I_{n+1}} \times \frac{H_{h}^{C_{n}} - H_{h}^{C_{n+1}}}{N}\right)$$
(2)

- $L_l^{C_n}$ = The number of loans when the interest rates of $I_1 \sim I_n$ based on the forecast π value are offered to borrowers and the interest rates based on the actual credit value of borrowers are $I_1 \sim I_n$.
- $H_l^{C_n}$ = The number of loans when the interest rates of $I_1 \sim I_n$ based on the forecast π value are offered to borrowers and the loans based on the actual credit value of borrowers are offered the interest rate of I_{n+1} to borrowers or yield a denied result.
- $L_h^{C_n}$ = The number of loans when the loans based on the forecast π value are offered the interest rate of I_{n+1} to borrowers or yield a denied result and interest rates based on the actual credit value of borrowers are $I_1 \sim I_n$.
- $H_h^{C_n}$ = The number of loans when the loan based on the forecast π value and the actual credit value of borrowers both yield an interest rate of I_{n+1} or there is a denied result.
- $R_{I_{1\sim n}}$ = The average interest rate of loans $I_1 \sim I_n$. $R_{I_{n+1}}$ = The average interest rate of loans I_{n+1} .

For this profit calculation, the accurate forecast profits and the incorrect forecast losses are summed up to calculate the profit maximization cut-off rate - that is, the optimum cut-off rate: C_n .

Term 1

 $\left(R_{I_{1\sim n}} \times \frac{L_{l}^{Cn}}{N}\right)$: The profits of accurate forecasts of borrowers' risk given interest rates of $I_{1} \sim I_{n}$. Since the number of loans offered at every interest rate interval for $I_{1} \sim I_{n}$ is not obtained in procedure 2, the product of the average

interest rate $(R_{I_{1\sim n}})$ and the probability $(\frac{L_l^{C_n}}{N})$ wi used for the expected returns for this outcome.

Term 2

$$\left[\left(1 + R_{I_{1 \sim n+1}} \right) \times \frac{\mu_{l}^{C_{n+1}}}{N} + \left(R_{I_{n+1}} - R_{I_{1 \sim n}} \right) \times \frac{\mu_{l}^{C_{n}} - \mu_{l}^{C_{n+1}}}{N} \right]:$$
 This term can be further

divided into the following two sections:

 $\left[\left(1 + R_{I_{1 \sim n+1}} \right) \times \frac{H_{l}^{C_{n+1}}}{N} \right]$: The default losses of underestimating borrowers' risk given interest rates of $I_{1} \sim I_{n+1}$.

 $\left[\left(R_{l_{n+1}} - R_{l_{1 \sim n}} \right) \times \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N} \right]$: The reduction in risk premium profits. Because

the forecast interest rate $(I_1 \sim I_n)$ is lower than the interest rate (I_{n+1}) based on the actual credit value of borrowers, the profits of the bank are reduced by $(R_{I_{n+1}} - R_{I_{1\sim n}})$.

Term 3

 $\left(R_{I_{1\sim(n+1)}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N}\right):$ This term can be further divided into

the two following sections:

 $\left(R_{I_{1\sim n+1}} \times \frac{L_{h}^{C_{n+1}}}{N}\right)$: The opportunity cost caused by incorrectly denying the loan applications given interest rates of $I_{1} \sim I_{n+1}$.

 $\left(R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N}\right)$: The opportunity cost caused by incorrectly denying the loan applications given interest rates of $I_{1} \sim I_{n}$; therefore, the average interest rate $\left(R_{I_{1-n}}\right)$ multiplied by the probability $\left(\frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N}\right)$ is the profit losses.

Term 4

 $\left(1 \times \frac{H_h^{C_{n+1}}}{N} + R_{I_{n+1}} \times \frac{H_h^{C_n} - H_h^{C_{n+1}}}{N}\right)$: This term can be further divided into two

sections:

 $\left(1 \times \frac{H_h^{C_{n+1}}}{N}\right)$: The losses avoided by accurate forecasts of borrowers' risk

and the rejection of these borrowers.

 $\left(R_{I_{n+1}} \times \frac{H_h^{C_n} - H_h^{C_{n+1}}}{N}\right)$: The profits of accurate forecasts of borrowers' risk given interest rates of I_{n+1} . The interest rate $\left(R_{I_{n+1}}\right)$ multiplied by the probability $\left(\frac{H_h^{C_n} - H_h^{C_{n+1}}}{N}\right)$ is the profits.

Procedure 3

Customers are divided into the markup groups $(I_1 \sim I_{n-1})$ and $(I_n \sim I_{n+1} + D)$ to calculate the optimum cut-off rate: C_{n-1} (Table 4).

Profit function C_{n-1} :

$$Profit = \left[R_{I_{1} \sim (n-1)} \times \frac{L_{l}^{Cn-1}}{N} \right] - \left[\left(1 + R_{I_{1} \sim (n+1)} \right) \times \frac{H_{l}^{Cn+1}}{N} + \left(R_{I_{n+1}} - R_{I_{1} \sim n} \right) \times \frac{H_{l}^{Cn} - H_{l}^{Cn+1}}{N} + \left(R_{I_{n}} - R_{I_{1} \sim (n-1)} \right) \times \frac{H_{l}^{Cn-1} - H_{l}^{Cn}}{N} \right] - \left[R_{I_{1} \sim (n+1)} \times \frac{L_{h}^{Cn+1}}{N} + R_{I_{1} \sim n} \times \frac{L_{h}^{Cn} - L_{h}^{Cn+1}}{N} + R_{I_{1} \sim (n-1)} \times \frac{L_{h}^{Cn-1} - L_{h}^{Cn}}{N} \right] + \left[1 \times \frac{H_{h}^{Cn+1}}{N} + R_{I_{n+1}} \times \frac{H_{h}^{Cn} - H_{h}^{Cn+1}}{N} + R_{I_{n}} \times \frac{H_{h}^{Cn-1} - H_{h}^{Cn}}{N} \right]$$
(3)

 $R_{I_{1}\sim(n-1)}$ = The average interest rate of loans $I_1 \sim I_{n-1}$. R_{I_n} = The interest rate of loan I_n .

For this profit calculation, the accurate forecast profits and the incorrect forecast losses are summed up to calculate the profit maximization cut-off rate - that is, the optimum cut-off rate: C_{n-1} .

Term 1

 $\left[R_{I_{1\sim(n-1)}} \times \frac{L_{l}^{C_{n-1}}}{N}\right]$: The profits of accurate forecasts of borrowers' risk given interest rates of $I_{1} \sim I_{n-1}$. The number of loans offered at every interest rate interval for $I_{1} \sim I_{n-1}$ is not obtained in procedure 3; therefore, the product of the average interest rate $(R_{I_{1\sim(n-1)}})$ and the probability $(\frac{L_{l}^{C_{n-1}}}{N})$ is used for the expected returns of this outcome.

Table 4Procedure Three

The actual credit value of borrowers (N = total loan amount) $(l) I_1 \sim I_{n-1} \qquad (l) I_n \sim I_{n+1} + D$ $(L) I_1 \sim I_{n-1} \qquad L_l^{C_{n-1}} \qquad L_h^{C_{n-1}}$ $(H) I_n \sim I_{n+1} + D \qquad H_l^{C_{n-1}} \qquad H_h^{C_{n-1}}$		Forecas	t π values
The actual credit value of borrowers $(H) I = I = D = U^{C_{n-1}} U^{C_{n-1}}$		$(l) I_1 {\sim} I_{n-1}$	(h) $I_n \sim I_{n+1} + D$
		L	п

1. In the previous procedure, the number C_n of the four quadrants $L_l^{C_n}$, $L_h^{C_n}$, $H_l^{C_n}$, and $H_h^{C_n}$ for the markups I_{n+1} is calculated; therefore, we list the sections separately. Additionally, the probability is 1, as expressed below.

$$\left[\frac{L_{l}^{C_{n-1}}}{N}\right] + \left[\frac{L_{h}^{C_{n+1}}}{N} + \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N} + \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N}\right] + \left[\frac{H_{l}^{C_{n+1}}}{N} + \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N} + \frac{H_{l}^{C_{n-1}} - H_{l}^{C_{n}}}{N}\right] + \left[\frac{H_{h}^{C_{n+1}} - H_{h}^{C_{n+1}}}{N} + \frac{H_{h}^{C_{n-1}} - H_{h}^{C_{n+1}}}{N}\right] + \left[\frac{H_{h}^{C_{n+1}} - H_{h}^{C_{n+1}}}{N} + \frac{H_{h}^{C_{n+1}} - H_{h}^{C_{n+1}}}{N}\right] + \left[\frac{H_{h}^{C_{n+1}} - H_{h}^{C_{n+1}}}{N$$

2. L and H: the actual credit value of borrowers

(L) $I_1 \sim I_{n-1}$: offered the interest-rate markups $I_1 \sim I_{n-1}$ based on the actual credit value of borrowers.

(*H*) $I_n \sim I_{n+1} + D$: offered the interest-rate markups $I_n \sim I_{n+1}$ based on the actual credit value of borrowers or a denied result based on the actual credit value of borrowers.

3. *l* and *h*: forecast π values.

(*l*) $I_1 \sim I_{n-1}$: when $\pi \leq C_{n-1}$, the interest-rate markups $I_1 \sim I_{n-1}$ are offered to borrowers.

(h) $I_n \sim I_{n+1}$: when $\pi > C_{n-1}$, the interest-rate markups $I_n \sim I_{n+1}$ are offered to borrowers or customers are denied a loan.

Term 2

$$\left[\left(1 + R_{I_{1} \sim n+1} \right) \times \frac{H_{l}^{Cn+1}}{N} + \left(R_{I_{n+1}} - R_{I_{1} \sim n} \right) \times \frac{H_{l}^{Cn} - H_{l}^{Cn+1}}{N} + \left(R_{I_{n}} - R_{I_{1} \sim (n-1)} \right) \times \frac{H_{l}^{Cn-1} - H_{l}^{Cn}}{N} \right]:$$

This term can be further divided into the following two sections:

$$\left[\left(1 + R_{I_{1 \sim n+1}} \right) \times \frac{H_{l}^{C_{n+1}}}{N} + \left(R_{I_{n+1}} - R_{I_{1 \sim n}} \right) \times \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N} \right]:$$
 The default losses and

reduced profits calculated in the previous procedure.

 $\left[\left(R_{I_n} - R_{I_{1\sim(n-1)}} \right) \times \frac{H_l^{C_{n-1}} - H_l^{C_n}}{N} \right]:$ The forecast interest rates $(I_1 \sim I_{n-1})$ are lower than the interest rate (I_n) based on the actual credit value of borrowers; therefore, the interest rate differential or differential interest rate $\left(R_{I_n} - R_{I_{1\sim(n-1)}} \right)$ multiplied by the probability $\left(\frac{H_l^{C_{n-1}} - H_l^{C_n}}{N} \right)$ is the reduced profits.

Term 3

$$\left[R_{I_{1\sim n+1}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim (n-1)}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N}\right]: \text{ This term can be}$$

further divided into the following two sections:

 $\left[R_{I_{1\sim n+1}} \times \frac{L_h^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_h^{C_n} - L_h^{C_{n+1}}}{N}\right]:$ The opportunity cost calculated in the

previous procedure.

 $\left[R_{I_{1\sim(n-1)}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N}\right]$: The opportunity cost caused by incorrectly denying the loan applications given interest rates of $(I_{1} \sim I_{n-1})$; therefore, the average interest rate $\left(R_{I_{1\sim(n-1)}}\right)$ multiplied by the probability $\left(\frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N}\right)$ is the profit losses.

Term 4

 $\left[1 \times \frac{H_h^{C_{n+1}}}{N} + R_{I_{n+1}} \times \frac{H_h^{C_n} - H_h^{C_{n+1}}}{N} + R_{I_n} \times \frac{H_h^{C_{n-1}} - H_h^{C_n}}{N}\right]:$ This term can be further divided into the following two sections:

 $\left[1 \times \frac{H_h^{C_{n+1}}}{N} + R_{I_{n+1}} \times \frac{H_h^{C_n} - H_h^{C_{n+1}}}{N}\right]:$ The profits and avoided losses calculated in the previous procedure.

 $\begin{bmatrix} R_{I_n} \times \frac{H_h^{C_{n-1}} - H_h^{C_n}}{N} \end{bmatrix}$: The interest rate R_{I_n} multiplied by the probability $\left(\frac{H_h^{C_{n-1}} - H_h^{C_n}}{N}\right)$ is the profits for this section.

Procedure 4~N

The above principle is used to identify the optimum cut-off rates of each interest markup interval: C_i , $i=(n-2)\sim 1$ (Table 5).

Profit function C_i

$$\begin{aligned} Profit &= \left[R_{I_{1-i}} \times \frac{L_{l}^{C_{l}}}{N} \right] - \left[\left(1 + R_{I_{1-n+1}} \right) \times \frac{H_{l}^{C_{n+1}}}{N} + \left(R_{I_{n+1}} - R_{I_{1-n}} \right) \times \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N} + \left(R_{I_{n}} - R_{I_{1-(n-1)}} \right) \right] \\ &\times \frac{H_{l}^{C_{n-1}} - H_{l}^{C_{n}}}{N} + \cdots + \left(R_{I_{l+1}} - R_{I_{1-i}} \right) \times \frac{H_{l}^{C_{i}} - H_{l}^{C_{i+1}}}{N} \right] - \left[R_{I_{1-n+1}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1-n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N} + R_{I_{1-(n-1)}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N} + R_{I_{1-i}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n-1}}}{N} + L_{h}^{C_{n-1}} + L_{h}^{C_{n-1$$

The profits of accurate forecasts minus the losses of risk premiums caused by underestimating loan risk and the profit losses caused by overestimating loan

Table 5Procedure Four ~N

		Fore	ecast π values
		(l) $I_1 \sim I_i$	$(h)(I_{i+1} \sim I_{n+1}) + D$
The actual credit value of borrowers	(L) $I_1 \sim I_i$	$L_l^{C_i}$	$L_h^{C_i}$
(N = overall loan amount)	$(H) (I_{i+1} \sim I_{n+1}) + D$	$H_l^{C_i}$	$H_h^{C_i}$
		-C: -C:	Ci i i i i i i

1. Using the previous procedure, the number of the four quadrants $L_l^{C_i}$, $L_h^{C_i}$, $H_l^{C_i}$, and $H_h^{C_i}$ for markups $(I_{i+1} \sim I_{n+1})$ is calculated; therefore, we list the calculated numbers or amounts separately. Additionally, the probability is 1, as expressed below.

$$\begin{split} \left[\frac{L_{l}^{C_{l}}}{N}\right] + \left[\frac{H_{l}^{C_{n+1}}}{N} + \dots + \frac{H_{l}^{C_{l+1}} - H_{l}^{C_{l+2}}}{N} + \frac{H_{l}^{C_{l}} - H_{l}^{C_{l+1}}}{N}\right] + \left[\frac{L_{h}^{C_{n+1}}}{N} + \dots + \frac{L_{h}^{C_{l+1}} - L_{h}^{C_{l+2}}}{N} + \frac{L_{h}^{C_{l}} - L_{h}^{C_{l+1}}}{N}\right] \\ + \left[\frac{H_{h}^{C_{n+1}}}{N} + \dots + \frac{H_{h}^{C_{l+1}} - H_{h}^{C_{l+2}}}{N} + \frac{H_{h}^{C_{l}} - H_{h}^{C_{l+1}}}{N}\right] = 1 \end{split}$$

2. L and H: the actual credit value of borrowers

(L) $I_1 \sim I_i$: offered the interest-rate markups $I_1 \sim I_i$ based on the actual credit value of borrowers.

(*H*) $I_{i+1} \sim I_{n+1}$: offered the interest-rate markups $I_{i+1} \sim I_{n+1}$ based on the actual credit value of borrowers or a denied result based on the actual credit value of borrowers.

3. *l* and *h*: forecast π values.

(1) $I_1 \sim I_i$: when $\pi \leq C_i$, the interest-rate markups $I_1 \sim I_i$ are offered to borrowers.

(h) $I_{i+1} \sim I_{n+1}$: when $\pi > C_i$, the interest-rate markups $I_{i+1} \sim I_{n+1}$ are offered to borrowers or customers denied a loan.

risk is the sum total of profit. Based on this calculation principle, we can identify the profit maximization and optimum cut-off rate, C_i . Through these procedures, we can identify the loan number of the interest-rate markups $I_{i+1} \sim I_{n+1}$ and the optimum cut-off rates $C_i \sim C_n$ of each interval based on the profit calculation principle. Using these procedures and models, we calculate the maximum profits of each interval and select the optimum cut-off rate of the interest rate intervals as the decision-making standard for the interest rate of approved loans.

Term 1

 $\left[R_{I_{1\sim i}} \times \frac{L_{i}^{C_{i}}}{N}\right]$: The profits of accurate forecasts of borrowers' risk given interest rates of $I_{1} \sim I_{i}$; the average interest rate $(R_{I_{1\sim i}})$ multiplied by the probability $(\frac{L_{i}^{C_{i}}}{N})$ is used for the expected returns for this outcome.

Term 2

$$\left[\left(1 + R_{l_{1} \sim n+1} \right) \times \frac{H_{l}^{C_{n+1}}}{N} + \left(R_{l_{n+1}} - R_{l_{1} \sim n} \right) \times \frac{H_{l}^{C_{n}} - H_{l}^{C_{n+1}}}{N} + \left(R_{l_{n}} - R_{l_{1} \sim (n-1)} \right) \times \frac{H_{l}^{C_{n-1}} - H_{l}^{C_{n}}}{N} + \frac{H_{l}^{C_{n-1}} - H_{l}^{C_{n}}}{$$

 $\dots + (R_{I_{l+1}} - R_{I_{1 \sim i}}) \times \frac{H_l^{\gamma_l} - H_l^{\gamma_{l+1}}}{N}$: This term can be further divided into the following two sections:

$$\left[\left(1 + R_{I_{1} \sim n+1}\right) \times \frac{H_{l}^{Cn+1}}{N} + \left(R_{I_{n+1}} - R_{I_{1} \sim n}\right) \times \frac{H_{l}^{Cn} - H_{l}^{Cn+1}}{N} + \left(R_{I_{n}} - R_{I_{1} \sim (n-1)}\right) \times \frac{H_{l}^{Cn} - 1 - H_{l}^{Cn}}{N} + \cdots \right]$$

The default losses and reduced profits calculated in the previous procedures.

 $\left[\left(R_{I_{i+1}} - R_{I_{1\sim i}} \right) \times \frac{H_l^{C_i} - H_l^{C_{i+1}}}{N} \right]:$ Because the forecast interest rate $(I_1 \sim I_i)$ is lower than the interest rate based (I_{i+1}) on the actual credit value of borrowers, the bank's profits are reduced; therefore, the interest rate differential or differential interest rate $\left(R_{I_{i+1}} - R_{I_{1\sim i}} \right)$ multiplied by the probability $\left(\frac{H_l^{C_i} - H_l^{C_{i+1}}}{N} \right)$ is the reduced interest differential.

Term 3

$$\left[R_{I_{1\sim n+1}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim (n-1)}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N} \dots + R_{I_{1\sim i}} \times \frac{L_{h}^{C_{i}} - L_{h}^{C_{i+1}}}{N}\right]: \text{ This}$$

term can be further divided into the following two sections:

 $\left[R_{I_{1\sim n+1}} \times \frac{L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim n}} \times \frac{L_{h}^{C_{n}} - L_{h}^{C_{n+1}}}{N} + R_{I_{1\sim (n-1)}} \times \frac{L_{h}^{C_{n-1}} - L_{h}^{C_{n}}}{N} \dots\right]: \text{ The opportunity cost}$

calculated in the previous procedures.

 $\left[R_{I_{1\sim i}} \times \frac{L_{h}^{C_{i}} - L_{h}^{C_{i+1}}}{N}\right]$: The opportunity cost caused by incorrectly denying the loan applications given interest rates of $(I_{1} \sim I_{i})$. Therefore, the average interest rate $(R_{I_{1\sim i}})$ multiplied by the probability $\left(\frac{L_{h}^{C_{i}} - L_{h}^{C_{i+1}}}{N}\right)$ is the profit losses.

Term 4

$$\left[1 \times \frac{H_h^{C_{n+1}}}{N} + R_{I_{n+1}} \times \frac{H_h^{C_n} - H_h^{C_{n+1}}}{N} + R_{I_n} \times \frac{H_h^{C_{n-1}} - H_h^{C_n}}{N} \dots + R_{I_{i+1}} \times \frac{H_h^{C_i} - H_h^{C_{i+1}}}{N}\right]: \text{ The avoided}$$

losses and profits of accurate forecasts of borrowers' risk.

The model of this study is about the interest rate and interest-rate markups of bank loans, based on the modification of the loan credit model of Dinh and Kleimeier (2007). Their model focuses on the credit model of the creditworthiness of bank loans. Although our model is based on their loan credit

model, we further address what is the optimal interest-rate markup for each bank loan that maximizes a bank's profit. The following is a comparison between our model and Dinh and Kleimeier's (2007).

This study's model first takes the profit function of the existing choice of profit and opportunity cost into account. Dinh and Kleimeier's (2007) model only considers the change in the opportunity cost of the profit function. This study's model is composed of four items: The profitability of the accepted loan, the losses of accepted bad debts, the losses of opportunity cost of unacceptable but should be good loans, and the opportunity profit when the banks correctly refuse to give a loan to defaulting customers to avoid bad debts. This study calculates the optimal cut-off rate carefully and comprehensively, as shown in formula (1).

In term 1 we calculate the profits that are from accurate forecasts of borrowers' risk given customers' interest rates. Term 2 presents the default losses of underestimating borrowers' risk given the principal and interest rates of customers. Term 3 is the opportunity cost caused by incorrectly denying the good loan applications given borrowers' interest rates. Finally, term 4 shows the losses avoided by accurately forecasting borrowers' default and the rejection of these borrowers' loan application. However, Dinh and Kleimeier (2007) only considered the margin profits from two types of opportunities cost changes in unaccepted bank loan. One is the profitable opportunity of correctly refusing the defaulting customers to avoid bad debts; the other is the loss of opportunity cost of unacceptable but should be good loans. We present the margin profit function of Dinh and Kleimeier (2007) as below:

$$\Delta \operatorname{profit} = \operatorname{cost} \operatorname{per} \operatorname{bad} \operatorname{loan} * B_b - \operatorname{benefit} \operatorname{per} \operatorname{good} \operatorname{load} * G_b$$
(5)

Term 3 in this present study is more like the second term of benefit per good loan * G_b in Dinh and Kleimeier's (2007) formula (5). Term 4 in this research is more like the first item in Dinh and Kleimeier's (2007) formula (5) of cost per bad loan * B_b . However, Dinh and Kleimeier (2007) did not take into account Term 1 and Term 2 of this paper, but only analyzed the margin profits of two types of opportunity cost changes in unaccepted bank loan. The difference between this paper and Dinh and Kleimeier's (2007) is that our model maximizes the profit function by taking into account both real earnings and losses in the event of bad loans.

The model of this research can also determine the level of interest rate and the interest-rate markups, while Dinh and Kleimeier's (2007) model cannot. This study's model can decide the interest rate and the corresponding cut-off rate by the profit function. Moreover, we can set the range of interest-rate markups and find the corresponding cut-off rate, which maximizes profit through the procedure in this study, while Dinh and Kleimeier (2007) only set up the cut-off rate to decide whether or not to accept the loan for the banks to obtain profit without the level of interest rate and the interest-rate markups.

This study's model also targets the interest-rate markups of bank loan, which are a part of the dynamic decision-making process. However, the model of Dinh and Kleimeier (2007) is the credit model of bank loans, which is a single decision factor for whether to grant credit or not. Our research model first determines whether to approve the loan or not and then decides the range of interest-rate markup according to the risk level. However, Dinh and Kleimeier (2007) simply analyzed the variation of opportunity cost to determine profit based on the possibility of default. Hence, their cut-off rate is only a simple default and non-default, and their model does not take into account the subsequent changes of interest-rate markups.

This study contrastingly considers the opportunity cost, the actual profit of the bank, and the process of determining interest-rate markups, which can accurately reflect the risk premium of the loan. Therefore, our model is the overall credit process, whereby we analyze whether to approve the loans or not, set the basis interest rate, and then gradually consider the interest-rate markups under increasing risk. This is a complete dynamic multi-credit decision-making process. Compared to Dinh and Kleimeier (2007), our model is evolutionary. Theirs is only a single factor decision model for initial credit and cannot be used to assess the risk based on all lending credit sample data in order to make a decision of interest-rate markups.

4. Research data and sample and variable choices

The purpose of this study is not to analyze the difference in the degree of risk aversion or the degree of interest rate caused by changes in the macroeconomic environment. However, the level of risk aversion and the difference in the interest rate both affect the magnitude of interest-rate markups.

First of all, we note the influence of the degree of risk aversion on the range of interest-rate markups. If the macroeconomic environment tends to be conservative, then the degree of risk aversion will increase and the risk premium naturally increases according to market mechanism. The interest-rate markups of the model, which entail the degree of the risk premium required by the bank, will become larger. On the contrary, if the macroeconomic environment tends to be positive, then the degree of risk aversion drops and the risk premium will decrease based on the market mechanism. Thus, the interest-rate markups of this study's model are relatively smaller.

Second, we take note of the influence of the difference in interest rate levels and the range of interest-rate markups. If the macroeconomic environment gets better, then the central bank will raise the level of market interest rate to cool the overall economy. In addition, the market will also increase forbearance to risk and reduce the degree of risk premium. Hence, the interest-rate markups of the model become lower. On the other hand, if the overall market economy deteriorates, then the central bank will reduce the market interest rate to stimulate the overall economy. Moreover, the market will reduce tolerance to risk, increase the degree of risk premium, and raise the overweight interest rate.

Cross-sectional analysis is required to assume that the spread rate of the default risk premium remains unchanged, such that the analysis will not be biased. However, in the empirical study, in order to increase the number of samples, we must collect the loan samples at different time points. However, the range of interest spreads at different time points will change according to the market economy environment and the degree of risk aversion, so that different interest spreads at different time points will generate different risk premium rates. With the same risk, the empirical samples will generate different interest-rate markups under different conditions, because the macroeconomic environment fluctuates significantly. However, the model in this study only examines the individual risks of the trustee and gives a reasonable interest-rate markup. Therefore, if the sample fluctuates according to a swing in the macroeconomic environment, then the system risk of the market will change dramatically. This will then change the degree of risk aversion and thus result in deviation.

Therefore, we need a relatively stable economic environment to analyze the feasibility of the optimal interest-rate markups considering given individual risk.

We believe that there are two directions that can reduce the bias of our results. The first is that we choose a short period when the interest rate is relatively stable (see Figure 1). Therefore, it can be assumed that the spread rate has not changed during the empirical sample period.

The second direction is that this study mainly analyzes the interest-rate markups of loans. Accordingly, as long as the regression analysis can capture those that are the risk variables of personal loans and when the risk variables have a significant positive or negative relationship, then there is monotonicity of the equation coefficient. When there exists a positive (negative) relationship, as long as the risk variables' values rise, the pitch in the markups increases (decreases). This empirical result supports our theoretical model.

The interest spreads of the default risk premium do vary from time to time due to differences in the economic situation. We cannot fully control the samples to be in the same economic situation, and this is the shortfall in this study. However, as long as these variables are significant, they can be used to capture the increase or decrease in the interest spreads, because of being monotonous.

In order to narrow the interest spreads, we set the period of this study to 2009-2015 - that is, during this period, the fluctuation of interest spreads is relatively stable. We thus dodge the interest spread changes that were more dramatic during the 2007-2008 financial crisis. Hence, the results of this study can be more convincing.

This study not only estimates the risk of loans, but also predicts the optimal interest-rate markups. In order to produce the most appropriate loan interest rate to facilitate competition with other banks, banks themselves do not have to bear too much risk. If the interest rate is too high, then although the bank can increase profits, it may also lose loyal customers. Conversely, if the interest rate is too low, then it will harm bank profit.

The samples used herein are customers who had been passed by the regional bank and do not contain failed customers. This study is likely to present the problem of sample selection bias mentioned by Greene (1998) and Banasik, Crook, and Thomas (2003). In other words, using only accepted applicants without the samples of unacceptable applicants and applying credit scoring



Note: Base rate is the average base rate of the five major banks in Taiwan. Source: Central Bank of the Republic of China (Taiwan).²

models to estimate the default loans will generate an estimate bias. Because the regional bank did not establish data on unacceptable applicants, the outcome of this study appears to have sample selection bias, because of the limited samples provided by the bank. However, Banasik *et al.* (2003) compared the results of samples including unacceptable applicants and samples excluding unacceptable applicants, showing only small differences between them.

The main purpose of this empirical model is to verify whether the theoretical model we derive is able to capture the difference between different loan interest-rate markups under different default risks. Therefore, if the bank has established data on unaccepted applicants, then we can add that data to estimate the default risks. Thus, we can make the interest-rate markups more accurate under different default risks. This would help achieve the goal of this study to improve the profitability of banks and reduce risk and make banks more competitive in the personal loan market.

² http://www.cbc.gov.tw/ct.asp?xItem=995&ctNode=523&mp=1.

Research data and samples

The research data for this study are credit-granted cases, including loans against collateral, unsecured loans, and consumer loans, sourced from the database of a bank in Taiwan. The research data cover 2009 to 2015. After eliminating observation values with insufficient variable information, 804 effective loan entries were derived; of which, 456 entries were regular customers and 348 entries were defaulting customers. The selection criteria for the research samples are as follows.

- (1) Held personal accounts and had applied for loans from the sample bank between 2009 and 2015.
- (2) The majority of loans are fiduciary loans and mortgages; of which, personal loans from cash cards and credit cards were excluded from fiduciary loans.
- (3) Regular customers refer to cases never defer or delay payment between 2009 and 2015.
- (4) Defaulting customers refer to cases transferred for collection and bad debt procedures.

The 25 variables used herein can be classified into four categories: personal characteristics, risk indicators, profit indicators, and others (Table 6). The variables of personal characteristics were provided by the applicants and reviewed and verified by the bank credit checking reviewers. The risk and profit indicators refer to the debt-repaying ability of the loan applicants at the time of loan application. The variables in the others category are those that cannot be classified into the other three categories. This classification can assist banks when reviewing loan applicants and evaluating profits and risks. Table 7 lists the descriptive statistics of the variables, and the results of the descriptive statistics are explained below.

Personal characteristics

The variables of personal characteristics include marital status, annual income, spouse's annual income, education levels, number of children, occupation, and years of employment (seven items). These variables are generally consistent with those employed by Crook *et al.* (1992), Dinh and Kleimerier (2007), and Steenackers and Goovaers (1989). They evaluated the

Name of variables	Definition	Categories of variables
Personal characteristics		
Marital status	Marital status of the borrower	0: Married 1: Single
Annual income	Annual income of the borrower	Actual income amount (thousand)
Spouse's annual income	Annual income of the borrower's spouse	Actual income amount (thousand)
Education levels	Highest education level of the borrower	 Doctoral program Graduate institute University (college) Senior high school Junior high school and below
Number of children Occupation Years of	Number of children of the borrower Job of the borrower Years of employment of the	 The actual number of children 1: Those responsible for listed/OTC companies and professionals (medicine, law, and accounting) 2: Civil servants, teachers, and managers and above of listed/OCT companies 3: The employees of listed/OTC companies, enterprise owners who conduct business with the bank, and customers with salary transfers to bank accounts 4: Other fixed salaries, independent proprietor or enterprise, or partnership enterprise owners 5: Others The actual years of employment
employment	borrower	The actual years of employment
Risk indicators	(T) 1	
The purpose of	The loan purpose	0: House purchase
loans		1: Repairs 2: Investment
		3: Compensation
		4: Auto purchase
		5: Others
Grace periods	The years for returning interest and not principal	The actual approved years
Collateral	Whether offering or setting collateral when applying for loans	0: No 1: Yes
Location of the collateral	Location of set collateral objects	0: None 1: City, township 2: County, village
Number of guarantors	Whether requesting or requisitioning guarantors when applying for loans	The actual number of requested or

Table 6Classification and definition of variables

Name of variables	Definition	Categories of variables
Salary certificates	Salary certificate of the borrower	0: No
		1: Yes
Credit status	Credit status of the borrower when	0: Normal
	applying for loans	1: Deferred payment within one month,
		but other payments are standard
		2: Deferred payment for one to three months or bounced checks but had been noted "paid off"
		3: Deferred payment or defaulted for over three months or have been refused by banks, but have returned to use
The period of credit-	Years that the borrower has loan	Actual years of credit-granting business
granting business	business with the loan application	retual years of creat granting susmess
8	bank	
Profit indicators		
Loan amount	Approved loan amount	Actual approved amount of loans
Loan period	Approved loan periods	Actual approved period of loans
Appropriation	The method for transferring loans to	
methods	the account of the borrower	1: Non-revolving
Repayment methods	The repayment methods after loan	0: Monthly interest payments
	approval	1: Principal-interest repayment
Cash cards	Whether the borrower held cash	0: No
	cards when applying for loans	1: Yes
Holding credit cards	Whether the borrower held credit	0: No
	cards when applying for loans	1: Yes
Others		
Deposit performance	Deposit amount of the borrower at the lending bank	Actual deposit performance value
Government	Policy-related loans	0: Policy-related loans
preferential policies		1: Non-policy-related loans
The period of active	Years that a borrower has had	Actual years of active bank account
bank accounts	deposit business with loan banks	business

 Table 6

 Classification and definition of variables (continued)

differences in characteristics and socioeconomic status of borrowers and found that the higher the socioeconomic status is, the lower the default rate.

The average annual income of borrowers is NT\$1,381,000, the standard deviation is NT\$3,341,000, and the difference between the maximum and minimum values is NT\$62,709,000, indicating that the incomes of the sample participants are drastically different and divergent. The average annual income of the borrowers' spouses is approximately NT\$235,000, the standard deviation is NT\$581,000, and the difference between the maximum and minimum values is

Table 7

The descriptive analytical results of the sample variables of personal loans

This table presents the variables of the four categories: personal characteristics, risk indicators, profit indicators, and others. The variables of personal characteristics are marital status, annual income, spouse's annual income, number of children, occupation, and years of employment (six items). The variables of risk indicators are the loan purpose, grace periods, collateral, location of the collateral, number of guarantors, salary certificates, credit status, and the period of credit-granting business. The salary certificate variables of profit indicators are loan amount, loan periods, appropriation methods, repayment methods, and the holding of credit cards and cash cards. The holding of credit cards and cash cards variable refers to whether a borrower owned those cards when applying for loans. The variables of others are levels for approving loans, deposit performance, government preferential policies, and the period of active bank accounts.

Variable	Mean	S. D.	Minimum 1	Maximum	Skewness	Kurtosis
Personal characteristics						
Marital status	0.317	0.475	0	1	0.855	-1.294
Annual income	1381.438	5341.128	0	62709	13.308	159.307
Spouse's annual income	235.719	581.312	0	9512	9.009	112.310
Education levels	2.871	0.952	0	4	-0.412	-1.129
Number of children	1.377	1.298	0	4	0.185	-1.013
Occupation	4.156	0.512	1	5	-0.689	5.903
Years of employment	7.951	4.315	1	25	0.981	0.881
Risk indicators						
Loan purpose	0.758	0.992	0	5	1.001	1.308
Grace periods	0.615	1.315	0	3	3.127	8.501
Collateral	0.899	0.567	0	1	-0.915	-0.790
Location of the collateral	1.315	0.708	0	2	0.133	0.357
Number of guarantors	0.991	0.713	0	4	0.681	2.311
Salary certificate	0.618	0.502	0	1	0.371	-1.901
Credit status	0.009	0.172	0	3	15.215	361.217
The period of	0.935	2.138	0	12	3.700	8.112
credit-granting business						
Profit indicators						
Loan amount		3557.913	50	25000	2.915	8.915
Loan period	11.752	6.389	1	30	-0.113	-1.318
Appropriation methods	0.917	0.138	0	1	-6.591	41.347
Repayment methods	0.811	0.372	0	1	-2.519	5.002
Cash cards	0.235	0.158	0	1	11.913	115.333
The holding of credit	0.491	0.529	0	1	1.735	-0.156
cards						
Others						
Level for approving loans	0.047	0.210	0	2	15.313	223.018
Deposit performance	38.109	103.418	0	1000	7.934	69.151
Government preferential	0.618	0.215	0	1	-1.319	0.721
policies						
The period of active bank	2.158	5.318	0	25	2.018	3.133
accounts						

NT\$9,512,000, indicating that the incomes of the sample participants' spouses are drastically different and divergent. This is equivalent to the condition of the borrowers. Furthermore, most of the customers have a low and very divergent annual income.

Risk indicators

Risk refers to the probability of default. High risk denotes an increased probability of default, which increases the interest-rate markups. The risk indicator variables include the loan purpose, grace periods, collateral, location of the collateral, number of guarantors, salary certificates, credit status, and the period of credit-granting business (eight items). The variables of collateral, location of the collateral, loan purpose, and the period of credit-granting business are consistent with those employed by Crook *et al.* (1992), Dinh and Kleimerier (2007), and Steenackers and Goovaers (1989). Credit status is an indicator for banks to evaluate the risk of borrowers. Borrowers with a poor credit status pose a greater risk. If the loan purpose is speculative, then default risk rates will increase and the interest-rate markup will also increase. The average period of credit-granting business is less than one year, the standard error is two years, and the difference between the maximum and minimum values exceeds one year, indicating that the customers have inconsistent periods of credit-granting business.

Profit indicators

The profit indicator variables include loan amount, loan periods, appropriation methods, repayment methods, cash cards, and the holding of credit cards (six items). The variable of loan amount is consistent with that reported by Carling *et al.* (2001). The higher the loan amount is, the higher the profits are. Different appropriation methods can affect the profit levels of banks. Upfront or one-time appropriation can yield more profit compared to split or revolving appropriation. Cash cards and credit cards indicate that borrowers understand or possess relatively new financial management concepts, which enables banks to promote new business, thereby increasing profits.

Others

The variables of the others category include the levels for approving loans, deposit performance, government preferential policies, and the period of active bank accounts (four items). Government preferential policies refer to policies developed by the bank to support government policies, including offering loans to minority groups. The government determines the interest rate for this type of loan. Because this type of loan is mostly an unsecured fiduciary loan, the risk is relatively high, which increases the default rate. Regarding secured mortgage loans, because banks cannot select customers and request extra risk premiums, the risk and default rates are higher than those of general secured loans. The variable of the period of active bank accounts is consistent with that employed by Crook *et al.* (1992), Dinh and Kleimerier (2007), and Steenackers and Goovaers (1989).

5. Empirical results

5.1 Mean value (F-test) results

Table 3 reports the mean values of the variables in this study for both "regular customers" and "defaulting customers". The table shows, at the 95% confidence level, that the variables, such as annual income, are related to the default, except for marital status and number of children in terms of personal characteristics. In the profit indicators, loan amount and loan periods are relevant to the default. In the category of others, there is also a certain degree of correlation between deposit performance and default. Table 8 presents the correlation of each variable to default.

Personal characteristics

The mean value between each variable and markup shows a decreasing or increasing situation. For example, the lower the "annual income" is, the higher the markup is. The shorter "years of employment" is, the greater the markup is. The shorter the "years of employment" is, the bigger the markup is. As for the

Table 8Mean value test (F-test) results

This table reports the mean value test (*F*-test) results of the variables for 'default' customers and five groups of 'normal' customers with varying loan interest rates; the normal customers and their corresponding markups are I_1 (1%), I_2 (2%), I_3 (2.5%), I_4 (3%), and I_5 (4.5%). The variables are classified into four categories: personal characteristics, risk indicators, profit indicators, and others. The variables of personal characteristics are marital status, annual income, spouse's annual income, education levels, number of children, and occupation. The variables of risk indicators are the loan purpose, grace periods, collateral, location of the collateral, number of guarantors, salary certificates, credit status, and the period of credit-granting business. The salary certificate variable refers to whether a borrower offered a salary certificate when applying for loans. The variables of profit indicators are loan amount, loan periods, appropriation methods, repayment methods, and the holding of credit cards and cash cards variable refers to whether a borrower owned the cards when applying for loans. The variables of others are levels for approving loans, deposit performance, government preferential policies, and the period of active bank accounts. * indicates significance at the 5% level.

Variable	Default	I_5	$I_{\scriptscriptstyle A}$	I_3	I_{2}	I_1	F-test
Personal characteristics				·			
Marital status	0.228	0.305	0.26			0.309	0.120
Annual income	1028.136	685.055	701.84			1698.378	1.874
Spouse's annual	180.021	170.190	248.52	7 260.851	1 196.208	219.709	0.178
income	2 502	2 002		a a cor	2 004	2 275	0.405*
Education levels	3.503	2.983	2.75			3.275	3.405*
Number of children	1.705	1.587	1.49			1.588	0.684
Occupation	4.081	3.798	3.59			4.048	1.113
Years of	6.705	5.045	5.75	8 9.015	6.587	8.075	6.247*
employment							
Risk indicators							
Loan purpose	0.730	1.005	1.395	0.801	0.587	0.794	7.133*
Grace periods	0.231	0.015	0.119	0.258	0.278	0.241	1.318
Collateral Location of the	$0.758 \\ 0.808$	$0.485 \\ 0.597$	$0.307 \\ 0.407$	$0.687 \\ 0.801$	$0.805 \\ 0.978$	$0.719 \\ 0.900$	14.899* 9.985*
collateral	0.808	0.597	0.407	0.801	0.978	0.900	9.985*
Number of guaranto	rs 1.208	0.987	0.851	0.507	1.034	1.098	17.864*
Salary certificates	0.289	0.551	0.298	0.612	0.355	0.365	8.236*
Credit status	0.001	0.002	0.104	0.003	0.001	0.005	2.851*
The period of credit- granting business	0.699	0.574	0.477	0.879	1.597	0.564	8.989*
Profit indicators							
Loan amount	2908.585	1203.854	1109.422	2578.151	2745.514	3950.875	4.984*
Loan periods	12.553	8.955	9.257	12.596	14.851	12.709	7.560*
Appropriation methods	0.893	1.001	1.000	0.984	0.945	0.964	0.354
Repayment methods	0.901	0.981	1.000	0.915	0.778	0.805	3.480*
The holding of cash cards	0.013	0.002	0.003	0.005	0.001	0.035	1.015
The holding of credit cards	0.064	0.185	0.133	0.215	0.147	0.2051	1.466
Others							
Levels for approvin	0.001	0.080	0.002	0.001	0.002	0.010	2.575*
loans							
Deposit performanc	78.853	6.540	7.557	41.105	30.219	25.349	1.084*
Government preferential policies	0.851	0.788	0.500	0.796	0.901	0.807	14.957*
The period of active bank accounts	1.680	1.701	0.777	2.513	3.180	2.555	2.647*

"education levels", the mean values appear irregularly, but in the markups between I_3 and I_4 , the mean values are lower than others, which imply that the borrowers' education levels are higher than other intervals.

Risk indicators

Each variable indicates a different mean value of markup in this indicator category. The mean value of the "loan purpose" is between 0.587 and 1.395, showing the purposes are mainly "house purchase" and "repairs". As for collateral, the mean values between I_1 and I_3 are higher than I_4 and I_5 , indicating that most of the borrowers provide collateral for lower interest rates. On the contrary, less people use collateral in the markups from I_4 to I_5 , which result in higher interest rates.

Profit indicators

In this category the mean value of loan amount is inversely correlated to the markup. With the average amount of the loan ranging from NT\$3,950,000 to NT\$1,109,000, higher mean values show fewer markups in I_1 , and longer mean values of the loan period in I_2 and I_3 mean lower interest rates. On the contrary, the markups between I_4 and I_5 are higher, and their loan periods are shorter than others. This result shows that the shorter the loan periods, the higher the markups.

Others

In this category "deposit performance" and "the period of active bank accounts" are the two significant variables. The mean value of "deposit performance" is around NT\$20,000 to NT\$35,000 between I_1 and I_3 and NT\$5,000 to NT\$6,000 between I_4 and I_5 . "The period of active bank accounts" also has the same situation that the longer the period is, the lower the markups. On the contrary, the shorter the opening period is, the higher the markup is. Combining the above two points, these two variables show correlations to the markups.

5.2 The results of logistic regression analysis

We conduct logistic regression analysis to identify which variables possess significant influence on defaults. The results indicate that certain variables in the four indicator categories all have significant influences on defaults (Table 9).

Personal characteristics

Among the seven personal characteristics variables, annual income, marital status, education levels, and occupations are correlated with defaults, of which, the influence of occupations is the most significant. This result is consistent with that reported by Crook *et al.* (1992). Civil servants and employees of listed companies have stable jobs and the higher their job position is, the lower the default probability is.

Risk indicators

The number of guarantors and the period of credit-granting business variables are more significant than the other risk indicator variables. The number of guarantors is a significant variable, because the majority of bank loans that require or requisition guarantors are fiduciary loans. We find that the longer the period of communication and business with the bank is, the more the banks understand the customers, and the lower the default probability is. This conclusion is consistent with that reported by Dinh and Kleimerier (2007).

Profit indicators

The loan periods and the holding of credit cards variables are more significant than the other profit indicator variables. A short loan period can lead to a relatively high principal and interest repayment amount per period, which increase the financial stress of the borrowers, resulting in higher default probability. This result is consistent with that reported by Dinh and Kleimerier (2007). Credit cards are a tool for deferring payment. Generally, people with credit cards have more debt (i.e., accounts payable) than those without credit cards, because they have more payables and expenses every month, which increase their default probability.

Table 9Logistic regression analysis

For this table, we use logistic regression analysis to examine whether the variables can forecast loan defaults. The variables in this study can be classified into four categories: personal characteristics, risk indicators, profit indicators, and others. The variables of personal characteristics are marital status, annual income, spouse's annual income, number of children, occupation, and years of employment (six items). The variables of risk indicators are the loan purpose, grace periods, collateral, location of the collateral, number of guarantors, salary certificates, credit status, and the period of credit-granting business. The salary certificate variable refers to whether a borrower offered a salary certificate when applying for loans. The variables of profit indicators are loan amount, loan periods, appropriation methods, repayment methods, and the holding of credit cards and cash cards variable refers to whether a borrower owned the cards when applying for loans. The variables of of the approving loans, deposit performance, government preferential policies, and the period of active bank accounts. B is the regression coefficient, and S.E. is the standard error.

Category	Variable	В	S. E.	Т	Significance
	Constant	-4.759	1.521	-3.375	0.001
Personal of	characteristics				
	Marital status	-0.308	0.221	-1.381	0.122
	Annual income	-0.002	0.001	-2.138	0.013**
	Spouse's annual income	-0.151	0.183	-1.216	0.137
	Education levels	0.222	0.181	1.708	0.128
	Number of children	0.018	0.075	0.118	0.670
	Occupation	0.609	0.173	2.480	0.080*
	Years of employment	0.030	0.019	1.218	0.201
Risk indic	cators				
	Loan purpose	0.001	0.180	0.021	0.813
	Grace periods	-0.163	1.180	-1.273	0.201
	Collateral	0.135	0.215	0.377	0.063*
	Location of the collateral	-0.063	0.275	-0.253	0.798
	Number of guarantors	0.355	0.181	2.404	0.030**
	Salary certificates	0.004	0.180	0.021	0.725
	Credit status	-15.132	12419.315	-0.001	0.900
	The period of credit-granting business	0.056	0.041	1.511	0.120
Profit ind	icators				
	Loan amount	-0.38	0.062	-0.738	0.061*
	Loan period	0.041	0.019	1.992	0.013**
	Appropriation methods	0.501	0.875	0.415	0.702
	Repayment methods	-0.721	0.518	-1.279	0.308
	Cash cards	-0.800	1.193	-0.571	0.668
	The holding of credit cards	-0.908	0.451	-4.089	0.057*
Others					
	Level for approving loans	-1.318	2.003	-0.519	0.399
	Deposit performance	0.001	0.001	1.318	0.028**
	Government preferential policies	-0.711		-2.718	0.013**
	The period of active bank accounts	-0.099		-2.913	0.009**

** denotes a 0.05 significance level; * denotes a 0.10 significance level.

Others

The variables of deposit performance, year (before or after the financial crisis in 2008), government preferential policies, and the period of active bank accounts have significant influences on defaults. Customers with superior deposit performance have a more superior financial status and lower default probability than customers with inferior deposit performance. The influence of year (before or after the financial crisis in 2008) is consistent with that reported by Avery *et al.* (2004) - that is, banks have become conservative on loan approval since the financial crisis in 2008. Banks are more rigorous when selecting customers, leading to a lower default probability when compared to before the 2008 financial tsunami. The influence of the period of active bank accounts is consistent with that reported by Crook *et al.* (1992) and Dinh and Kleimerier (2007). The more years the banks have interacted with customers, the more the banks understand them, which improves the comprehensiveness of information collection during loan application or awarding periods, thereby increasing the forecast accuracy and reducing default probability.

5.2 The optimum cut-off rate of interest-rate markups

Based on the statistical analysis results, we identify the borrowers with relatively high default probability, which can reduce the loan risk and improve the operational performance of the bank. However, determining the optimum interest-rate markup is the key factor for banks' loan business. After excluding the defaulting customers from the samples, we take 456 regular customers as the sample to calculate profits. We use the benchmark interest rate of 1.155% of the sample bank as the loan interest rate. Based on the current distribution of personal loan interest rates for the sample bank, the interest rates are divided into five intervals (I_1 , I_2 , I_3 , I_4 , and I_5) with the markup rates of 1%, 2%, 2.5%, 3%, and 4.5%. We employ the previous determined linear formula as follows:

$$Z_j = W_x = w_0 + w_1 x_{j1} + w_2 x_{j2} + \dots + w_k x_{jk}$$
 and $\pi_j = \frac{1}{1 + e^{-(W'_x)}}$

We calculate the π value of each borrower to determine the pricing of interest-rate markups (Table 10).

Based on the markup intervals, we substitute the profit calculation models for the profit functions 1 to 4 to calculate the profit maximization cut-off rates -

F					Unit: %
Cut-off Rate	I	C_1	C_2	C_3	C_4
Intervals	I_1	I_2	I_3	I_4	I_5
Markups Interest rates	1% 2.155%	2% 3.155%	2.5% 3.655%	3% 4.155%	.5% 5.655%

Table 10Markup Intervals and Interest Rates

that is, the optimum cut-off rates: $C_4 \sim C_1$. Table 11 shows the increase in bank profits from the cut-off rates for each interest-rate markup interval.

The interest-rate markups of the samples are more concentrated in certain interest rate ranges, which mean most of the interest-rates markups of the samples fall within certain spreads, resulting in less samples of higher and lower interest rates. The main purpose of this paper is to verify the feasibility of our interest-rate markup model. The four quadrants of the profit function in this model are composed of profit and opportunity cost in order to make easier analysis of cut-off rates and more equal cut-off samples in each quadrant. Therefore, the probability value in the profit function can be changed accordingly in the four quadrants of this model, so that the profit function changes in order to achieve maximum profit. We then find the cut-off rate for this study and use the markup rates of 1%, 2%, 2.5%, 3%, and 4.5%, in order to obtain the existences of real samples in each quadrant in their corresponding cute-off rate and hence observe the evolution from the change of interest intervals.

The monotonicity and changing direction basically do not change. The higher the risk is, the higher the interest rate. However, the interest-rate markup in the risk premium is different under different risk levels. Therefore, setting different ranges of interest-rate markups does not change the monotonicity that higher risk means higher interest-rate markups.

To collate the bank's comprehensive loan data, we divide the customers into credit-granted customers and denied customers. However, because the proportion of defaulting customers in the samples is excessive, the cut-off rates of the various interest-rate markups concentrate in certain areas for credit-granted

Table 11The optimum cut-off rates

This table presents the profits of the bank within each cut-off rate. C_1 , C_2 , C_3 , and C_4 are the cut-off rates of the five markup intervals. L_l is the number of loans that had equivalent low forecast and actual interest rates. L_h is the number of loans that had a forecast interest rate greater than the actual interest rate. H_l is the number of loans that had a forecast interest rate lower than the actual interest rate. H_h is the number of loans that had equivalent high forecast and actual interest rate. The number of loans that had equivalent high forecast and actual interest rate. The number of loans that had equivalent high forecast and actual interest rates. Profit is the sum of the profits of each cut-off rate. Probability is the number of loans in the various conditions divided by the total number of loans.

			<i>C</i> ₁					<i>C</i> ₂					<i>C</i> ₃					<i>C</i> ₄		
Cut-	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit
off		(Proba	bility)				(Probal	bility)				(Proba	bility)				(Proba	bility)		
0.99	220	0	236	0	0.0056	320	0	136	0	0.0148	388	0	68	0	0.0224	424	0	32	0	0.0260
	(0.48)	(0.00)	(0.52)	(0.00)		(0.70)	(0.00)	(0.30)	(0.00)		(0.85)	(0.00)	(0.15)	(0.00)		(0.93)	(0.00)	(0.07)	(0.00)	
0.90	220	0	233	3	0.0058	320	0	131	5	0.0154	388	0	64	4	0.0230	424	0	30	2	0.0264
	(0.48)	(0.00)	(0.51)	(0.01)		(0.70)	(0.00)	(0.29)	(0.01)		(0.85)	(0.00)	(0.14)	(0.01)		(0.93)	(0.00)	(0.06)	(0.01)	
0.80	220	0	226	10	0.0065	320	0	124	12	0.0146	385	0	59	12	0.0237	421	2	22	11	0.0277
	(0.48)	(0.00)	(0.50)	(0.02)		(0.70)	(0.00)	(0.27)	(0.03)		(0.85)	(0.00)	(0.13)	(0.02)		(0.92)	(0.01)	(0.05)	(0.02)	
0.70	218	1	212	25	0.0078	318	5	114	19	0.0167	379	8	47	22	0.0242	405	20	18	13	0.0260
	(0.48)	(0.00)	(0.47)	(0.05)		(0.70)	(0.01)	(0.25)	(0.04)		(0.83)	(0.02)	(0.10)	(0.05)		(0.89)	(0.04)	(0.04)	(0.03)	
0.60	210	13	188	45	0.0088	305	17	101	33	0.0168	365	25	38	28	0.0231	382	47	12	15	0.0234
	(0.46)	(0.03)	(0.41)	(0.10)		(0.67)	(0.04)	(0.22)	(0.07)	_	(0.80)	(0.06)	(0.08)	(0.06)		(0.84)	(0.10)	(0.03)	(0.03)	
0.59	208	13	186	49	0.0091	304	18	95	39	0.0173	320	78	28	30	0.0175	380	50	11	15	0.0231
	(0.45)	(0.03)	(0.41)	(0.11)		(0.66)	(0.04)	(0.21)	(0.09)		(0.70)	(0.17)	(0.06)	(0.07)		(0.84)	(0.11)	(0.002)	(0.03)	
0.50	195	32	159	75	0.0098	266	55	82	53	0.0145	280	130	14	32	0.0123	324	106	10	16	0.0159
0.40	(0.43)	(0.07)	(0.34)	(0.16)	0.0405	(0.58)	(0.12)	(0.18)	(0.12)	0.0400	(0.61)	(0.29)	(0.03)	(0.07)		(0.71)	(0.23)	(0.02)	(0.04)	0.0040
0.40	164	60	120	112	0.0107	217 (0.48)	105	64	70	0.0108	255	150 (0.33)	11 (0.02)	40 (0.09)	0.0136	255 (0.56)	175 (0.38)	9	17 (0.04)	0.0069
0.37	(0.36) 160	(0.13)	(0.26) 99	(0.25)	0.0115	200	(0.23)	(0.14) 67	(0.15) 70	0.0089	(0.56) 185	(0.33)	10	(0.09)	0.0016	(0.56)	(0.38)	(0.02)	(0.04)	-0.0063
0.57	(0.35)	(0.15)	(0.22)	(0.28)	0.0115	(0.44)	(0.26)	(0.15)	(0.15)	0.0089	(0.41)	(0.48)	(0.02)	(0.09)	0.0010	(0.34)	(0.61)	4 (0.01)	(0.04)	-0.0005
0.30	117	103	97	139	0.0087	158	156	61	81	0.0054	138	268	8	42	-0.0042	145	290	3	18	-0.0074
0.50	(0.26)	(0.23)	(0.21)	(0.30)	0.0007	(0.35)	(0.34)	(0.14)	(0.17)	0.0004	(0.30)	(0.59)	(0.02)	(0.09)	0.0042	(0.32)	(0.63)	(0.01)	(0.04)	0.0074
0.20	91	131	76	158	0.0078	125	195	46	90	0.0024	136	268	6	46	-0.0039	99	337	2	18	-0.0135
0.20	(0.20)	(0.29)	(0.17)	(0.34)	0.0070	(0.27)	(0.43)	(0.10)	(0.20)	0.0021	(0.30)	(0.59)	(0.01)	(0.10)	0.0000	(0.21)	(0.74)	(0.01)	(0.04)	0.0155
0.10	62	157	62	175	0.0067	95	238	25	98	-0.0005	95	309	4	48	-0.0088	80	356	1	19	-0.0158
	(0.14)	(0.34)	(0.14)	(0.38)		(0.21)	(0.52)	(0.05)	(0.22)		(0.21)	(0.68)	(0.01)	(0.10)		(0.17)	(0.78)	(0.01)	(0.04)	
0.01	38	180	25	213	0.0080	51	280	16	109	-0.0043	50	351	0	55	-0.0135	42	394	0	20	-0.0207
	(0.08)	(0.39)	(0.05)	(0.48)		(0.11)	(0.61)	(0.04)	(0.24)		(0.11)	(0.77)	(0.00)	(0.12)		(0.09)	(0.87)	(0.00)	(0.004)	

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customers under the condition of normal risk. This prevents the empirical model of this study from determining the optimum interest-rate markups under various risk types. Therefore, Procedure 1 is not performed. We then move directly to Procedure 2 and classify credit-granted customers to calculate the optimum cut-off rates.

Procedure 1: Calculate the optimum cut-off rate C_4

We divide the customers into two categories, I_5 (with a markup of 4.5%) and $I_1 \sim I_4$ (with a markup of 1% to 3%), to calculate the optimum cut-off rate: C_4 (Table 12).

Profit function 1

$$Profit = \left[R_{I_{1\sim4}} \times \frac{L_{l}^{C_{4}}}{N}\right] - \left[\left(R_{I_{5}} - R_{I_{1\sim4}}\right) \times \frac{H_{l}^{C_{4}}}{N}\right] - \left[R_{I_{1\sim4}} \times \frac{L_{h}^{C_{4}}}{N}\right] + \left[R_{I_{5}} \times \frac{H_{h}^{C_{4}}}{N}\right].$$
(1)

The profits from accurate forecasts minus the losses of incorrect forecasts are the bank profits.

N = 456

Table 12Calculate the optimum cut-off rate C_4

		Forecas	t π values
		(<i>l</i>) $I_1 \sim I_4$	(h) I_5
	(<i>L</i>) $I_1 \sim I_4$	$L_l^{C_4}$	$L_h^{C_4}$
The actual credit value of borrowers	(<i>H</i>) <i>I</i> ₅	$H_l^{C_4}$	$H_h^{C_4}$

L and *H*: The actual credit value of borrowers.

(L) $I_1 \sim I_4$: Based on the actual credit value of the borrowers, the interest-rate markups of $I_1 \sim I_4$ (an interest rate ranging between 2.155% and 4.155%) are offered.

(*H*) I_5 : Based on the actual credit value of the borrowers, the interest-rate markups of I_5 (the interest rate 5.655%) are offered.

l and *h*: forecast π values.

(*l*) $I_1 \sim I_4$: when $\pi \leq C_4$, the interest-rate markups $I_1 \sim I_4$ (an interest rate ranging between 2.155%) and 4.155%) are offered to borrowers.

(h) I_5 when $\pi > C_4$, the interest-rate markups I_5 (the interest rate 5.655%) are offered to borrowers.
- $L_l^{C_4}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are $I_1 \sim I_4$ (2.155% ~ 4.155%).
- $H_l^{C_4}$ = The number of loans when the forecast interest rates of the π value are $I_1 \sim I_4$ (2.155%~4.155%) and the interest rate based on the actual credit value of borrowers is $I_5(5.655\%)$.
- $L_h^{C_4}$ = The number of loans when the forecast interest rates of the π value are $I_5(5.655\%)$ and the interest rate based on the actual credit value of borrowers is $I_1 \sim I_4$ (2.155% ~ 4.155%).
- $H_h^{C_4}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are $I_5(5.655\%)$.
- $R_{I_{1\sim4}}$ = The average interest rate (2.834%) of loans $I_1 \sim I_4$.

 R_{I_5} = The interest rate (2.834%) of loans I_5 .

The profit calculation formula can be rewritten as follows:

$$Profit = \left[(2.834\%) \times \frac{L_l^{C_4}}{456} \right] - \left[(2.821\%) \times \frac{H_l^{C_4}}{456} \right] - \left[(2.834\%) \times \frac{L_h^{C_4}}{456} \right] + \left[(5.655\%) \times \frac{H_h^{C_4}}{456} \right]$$

The results indicate that when $L_l^{C_4}$, $L_h^{C_4}$, $H_l^{C_4}$, and $H_h^{C_4}$ are (421, 2, 22, 11), the maximum profit is 0.0277. Therefore, a cut-off rate (0.80) is the optimum cut-off rate: C_4 (Table 11).

Procedure 2: Calculate the optimum cut-off rate C_3

We divide the customers into $(I_1 \sim I_3$ with the markups of 1%, 2%, and 2.5%) and $(I_4 \sim I_5$ with the markups of 3% and 4.5%) to calculate the optimum cut-off rate, C_3 (Table 13).

Profit function 2

$$Profit = \left[R_{I_{1\sim3}} \times \frac{L_{l}^{C_{3}}}{N} \right] - \left[\left(R_{I_{5}} - R_{I_{1\sim4}} \right) \times \frac{H_{l}^{C_{4}}}{N} + \left(R_{I_{4}} - R_{I_{1\sim3}} \right) \times \frac{H_{l}^{C_{3}} - H_{l}^{C_{4}}}{N} \right] - \left[R_{I_{1\sim4}} \times \frac{L_{h}^{C_{4}}}{N} + R_{I_{1\sim4}} \times \frac{H_{h}^{C_{3}} - H_{h}^{C_{4}}}{N} \right]$$

$$R_{I_{1\sim3}} \times \frac{L_{h}^{C_{3}} - L_{h}^{C_{4}}}{N} + \left[R_{I_{5}} \times \frac{H_{h}^{C_{4}}}{N} + R_{I_{4}} \times \frac{H_{h}^{C_{3}} - H_{h}^{C_{4}}}{N} \right]$$

$$(2)$$

 $L_l^{C_3}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are $I_1 \sim I_3$ (2.155% ~ 3.655%).

Calculate th	The optimum cut-off rate C_3 Forecast π values (l) $I_1 \sim I_3$ (h) $I_4 \sim I_5$ (L) $I_1 \sim I_3$ $L_l^{C_3}$ $L_h^{C_3}$ (H) $I_4 \sim I_5$ $H_l^{C_3}$ $H_h^{C_3}$						
	Forecast π values						
		(<i>l</i>) $I_1 \sim I_3$	(<i>h</i>) $I_4 \sim I_5$				
	(<i>L</i>) $I_1 \sim I_3$	$L_l^{C_3}$	$L_h^{C_3}$				
The actual credit value of borrowers	(<i>H</i>) $I_4 \sim I_5$	$H_l^{C_3}$	$H_h^{C_3}$				

Table 13 Calculate the optimum cut-off rate C

L and H: The actual credit value of borrowers.

(L) $I_1 \sim I_3$: Based on the actual credit value of the borrowers, the interest-rate markups of $I_1 \sim I_3$ (an interest rate ranging between 2.155% and 3.655%) are offered.

(*H*) $I_4 \sim I_5$: Based on the actual credit value of the borrowers, the interest-rate markups of I_4 and I_5 (the interest rate 4.155% \sim 5.655%) are offered.

l and *h*: forecast π values.

(*l*) $I_1 \sim I_3$: when $\pi \leq C_3$, the interest-rate markups $I_1 \sim I_3$ (an interest rate ranging between 2.155% and 3.655%) are offered to borrowers.

(h) $I_4 \sim I_5$: when $\pi > C_3$, the interest-rate markups I_4 and I_5 (the interest rates 4.155% and 5.655%) are offered to borrowers.

- $H_l^{C_3}$ = The number of loans when the forecast interest rates of the π value are $I_1 \sim I_3$ (2.155%~3.655%) and the interest rates based on the actual credit value of borrowers are I_4 and I_5 (4.155%, 5.655%).
- $L_h^{C_3}$ = The number of loans when the forecast interest rates of the π value are I_4 and I_5 (4.155% and 5.655%) and the interest rate based on the actual credit value of borrowers is $I_1 \sim I_3$ (2.155% ~ 3.655%).
- $H_h^{C_3}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are I_4 and I_5 (4.155%, 5.655%).
- $R_{I_{1\sim3}}$ = The average interest rate (2.710%) of loans $I_1 \sim I_3$.
- R_{I_4} = The interest rate (4.155%) of loans I_4 .

In this phase, the numbers of I_5 and I_4 are classified and categorized. The profit calculation formula can be rewritten as follows:

$$\operatorname{Profit} = \left[(2.710\%) \times \frac{L_l^{C_3}}{456} \right] - \left[(2.821\%) \times \frac{22}{456} + (1.445\%) \times \frac{H_l^{C_3} - 22}{456} \right] - \left[(2.834\%) \times \frac{2}{456} + (2.710\%) \times \frac{L_h^{C_3} - 2}{456} \right] + \left[(5.655\%) \times \frac{11}{456} + (4.155\%) \times \frac{H_h^{C_3} - 11}{456} \right]$$

The profit of accurate forecasts $(L_l^{C_3} \text{ and } H_h^{C_3})$ minus the premium losses of underestimated loan risk $(H_l^{C_3})$ and the profit losses of overestimated loan risk is the sum total of profit. When $L_l^{C_3}$, $L_h^{C_3}$, $H_l^{C_3}$, and $H_h^{C_3}$ are (379, 8, 47, 22), the maximum profit is 0.0242, and the cut-off rate is 0.70 - that is, the optimum cut-off rate C_3 (Table 11).

Procedure 3: Calculate the optimum cut-off rate C_2

We divide the customers into $(I_1 \text{ and } I_2 \text{ with the markups of 1\% and 2\%})$ and $(I_3, I_4, \text{ and } I_5 \text{ with the markups of 2.5\%, 3\%, and 4.5\%})$ to calculate the optimum cut-off rate C_2 (Table 14).

Profit function 3

$$Profit = \left[R_{I_{1\sim2}} \times \frac{L_{l}^{C_{2}}}{N} \right] - \left[\left(R_{I_{5}} - R_{I_{1\sim4}} \right) \times \frac{H_{l}^{C_{4}}}{N} + \left(R_{I_{4}} - R_{I_{1\sim3}} \right) \times \frac{H_{l}^{C_{3}} - H_{l}^{C_{4}}}{N} + \left(R_{I_{3}} - R_{I_{1\sim2}} \right) \times \frac{H_{l}^{C_{2}} - H_{l}^{C_{3}}}{N} \right] - \left[R_{I_{1\sim4}} \times \frac{L_{h}^{C_{4}}}{N} + R_{I_{1\sim3}} \times \frac{L_{h}^{C_{3}} - L_{h}^{C_{4}}}{N} + R_{I_{1\sim2}} \times \frac{L_{h}^{C_{2}} - L_{h}^{C_{3}}}{N} \right] + \left[R_{I_{5}} \times \frac{H_{h}^{C_{4}}}{N} + R_{I_{4}} \times \frac{H_{h}^{C_{3}} - H_{h}^{C_{4}}}{N} + R_{I_{3}} \times \frac{H_{h}^{C_{2}} - H_{h}^{C_{3}}}{N} \right]$$
(3)

Table 14Calculate the optimum cut-off rate C_2

	Forecast π values			
	(<i>l</i>) $I_1 \sim I_2$	(<i>h</i>) $I_3 \sim I_5$		
(<i>L</i>) $I_1 \sim I_2$	$L_l^{C_2}$	$L_h^{C_2}$		
(<i>H</i>) $I_3 \sim I_5$	$H_l^{C_2}$	$H_h^{C_2}$		
		(<i>l</i>) $I_1 \sim I_2$ (<i>L</i>) $I_1 \sim I_2$ (<i>L</i>) $L_1 \sim I_2$		

L and *H*: The actual credit value of borrowers.

(L) $I_1 \sim I_2$: Based on the actual credit value of the borrowers, the interest-rate markups of $I_1 \sim I_2$ (an interest rate ranging between 2.155% and 3.155%) are offered.

l and *h*: forecast π values.

(*l*) $I_1 \sim I_2$: when $\pi \leq C_2$, the interest-rate markups $I_1 \sim I_2$ (an interest rate ranging between 2.155% and 3.155%) are offered to borrowers.

(*h*) $I_3 \sim I_5$: when $\pi > C_2$, the interest-rate markups I_3 , I_4 , and I_5 (the interest rate 3.655%, 4.155%, 5.655%) are offered.

 $⁽H)I_3 \sim I_5$: Based on the actual credit value of the borrowers, the interest-rate markups of I_3 , I_4 , and I_5 (the interest rate 3.655%, 4.155%, 5.655%) are offered.

- $L_l^{C_2}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are $I_1 \sim I_2$ (2.155% ~ 3.155%).
- $H_l^{C_2}$ = The number of loans when the forecast interest rates of the π value are $I_1 \sim I_2$ (2.155%~3.155%) and the interest rate based on the actual credit value of borrowers are I_3 , I_4 , and I_5 (the interest rate 3.655%, 4.155%, 5.655%)
- $L_h^{C_2}$ = The number of loans when the forecast interest rates of the π value are I_3 , I_4 , and I_5 (the interest rate 3.655%, 4.155%, 5.655%) and the interest rate based on the actual credit value of borrowers is $I_1 \sim I_2$ (2.155% ~ 3.155%).
- $H_h^{C_2}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are I_3 , I_4 , and I_5 (3.655%, 4.155%, 5.655%).
- $R_{I_{1\sim2}}$ = The average interest rate (2.476%) of loans $I_1 \sim I_2$. R_{I_3} = The interest rate (3.655%) of loans I_3 .

In this phase, the numbers of I_5 , I_4 , and I_3 are classified and categorized. The profit calculation formula can be rewritten as follows:

$$Profit = \left[(2.476\%) \times \frac{L_{l}^{C_2}}{456} \right] - \left[(2.821\%) \times \frac{22}{456} + (1.445\%) \times \frac{47-22}{456} + (1.176\%) \times \frac{H_{l}^{C_2}-47}{456} \right] - \left[(2.834\%) \times \frac{2}{456} + (2.71\%) \times \frac{8-2}{456} + (2.476\%) \times \frac{L_{h}^{C_2}-8}{456} \right] + \left[(5.655\%) \times \frac{11}{456} + (4.155\%) \times \frac{22-11}{456} + (3.655\%) \times \frac{H_{h}^{C_2}-22}{456} \right]$$

The profit of accurate forecasts on loans $(L_l^{C_2} \text{ and } H_h^{C_2})$ minus the premium losses of underestimated loan risk $(H_l^{C_2})$ and the customer losses of overestimated loan risk $(L_h^{C_2})$ is the sum total of profit. When $L_l^{C_2}$, $H_l^{C_2}$, $L_h^{C_2}$ and $H_h^{C_2}$ are (304, 18, 95, 39), the maximum profit is obtained (0.0173), and the cut-off rate is 0.59 - that is, the optimum cut-off rate, C_2 (Table 11).

Procedure 4: Calculate the optimum cut-off rate C_1

We finally divide the customers into $(I_1$ with the markups of 1%) and $(I_2, I_3, I_4, and I_5$ with the markups of 2%, 2.5%, 3%, and 4.5%) to calculate the optimum cut-off rate C_1 (Table 15).

Table 15Calculate the optimum cut-off rate C_1

		F	$L_{h}^{C_{1}}$	
		$(l) \ I_1$	(h) $I_2 + I_3 + I_4 + I_5$	
	(L) I_1	$L_l^{C_1}$	$L_h^{C_1}$	
The actual credit value of borrowers	(<i>H</i>) $I_2 + I_3 + I_4 + I_5$	$H_l^{C_1}$	$H_h^{C_1}$	

L and H: The actual credit value of borrowers.

(L) I_1 : Based on the actual credit value of the borrowers, the interest-rate markups of I_1 (the interest rate ranging 2.155%) are offered.

(*H*) $I_2 + I_3 + I_4 + I_5$: Based on the actual credit value of the borrowers, the interest-rate markups of I_2 , I_3 , I_4 , and I_5 (the interest rate 3.155%, 3.655%, 4.155%, 5.655%) are offered.

l and *h*: forecast π values.

(*l*) I_1 : when $\pi \leq C_1$, the interest-rate markups I_1 (the interest rate 2.155%) are offered to borrowers.

(h) $I_2 + I_3 + I_4 + I_5$: when $\pi > C_1$, the interest-rate markups I_2 , I_3 , I_4 , and I_5 (the interest rate 3.155%, 3.655%, 4.155%, 5.655%) are offered.

Profit function 4

$$Profit = \left[R_{I_{1}} \times \frac{L_{l}^{C_{1}}}{N}\right] - \left[\left(R_{I_{5}} - R_{I_{1} \sim 4}\right) \times \frac{H_{l}^{C_{4}}}{N} + \left(R_{I_{4}} - R_{I_{1} \sim 3}\right) \times \frac{H_{l}^{C_{3}} - H_{l}^{C_{4}}}{N} + \left(R_{I_{3}} - R_{I_{1} \sim 2}\right) \times \frac{H_{l}^{C_{2}} - H_{l}^{C_{3}}}{N} + \left(R_{I_{2}} - R_{I_{1}}\right) \times \frac{H_{l}^{C_{1}} - H_{l}^{C_{2}}}{N}\right] - \left[R_{I_{1} \sim 4} \times \frac{L_{h}^{C_{4}}}{N} + R_{I_{1} \sim 3} \times \frac{L_{h}^{C_{3}} - L_{h}^{C_{4}}}{N} + R_{I_{1} \sim 2} \times \frac{L_{h}^{C_{2}} - L_{h}^{C_{3}}}{N} + R_{I_{1}} \times \frac{L_{h}^{C_{1}} - L_{h}^{C_{2}}}{N}\right] + \left[R_{I_{5}} \times \frac{H_{h}^{C_{4}}}{N} + R_{I_{4}} \times \frac{H_{h}^{C_{3}} - H_{h}^{C_{4}}}{N} + R_{I_{3}} \times \frac{H_{h}^{C_{2}} - H_{h}^{C_{3}}}{N} + R_{I_{2}} \times \frac{H_{h}^{C_{1}} - H_{h}^{C_{2}}}{N}\right]$$
(4)

 $L_l^{C_1}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are I_1 (2.155%).

- $H_l^{C_1}$ = The number of loans when the forecast interest rates of the π value are I_1 (2.155%) and the interest rates based on the actual credit value of borrowers are I_2 , I_3 , I_4 , and I_5 (the interest rate 3.155%, 3.655%, 4.155%, and 5.655%).
- $L_h^{C_1}$ = The number of loans when the forecast interest rates of the π value are I_2 , I_3 , I_4 , and I_5 (the interest rate 3.155%, 3.655%, 4.155%, and 5.655%) and the interest rate based on the actual credit value of borrowers is I_1 (2.155%).
- $H_h^{C_1}$ = The number of loans when the forecast interest rate of the π value and the interest rate based on the actual credit value of borrowers are I_2 , I_3 , I_4 , and I_5 (the interest rate

3.155%, 3.655%, 4.155%, and 5.655%).

In this phase, the numbers of I_5 , I_4 , I_3 , I_2 , and I_1 are completely classified and categorized. Because the numbers of I_5 , I_4 , I_3 , and I_2 are classified, we use the interest rates R_{I_5} , R_{I_4} , R_{I_3} , and R_{I_2} (5.655%, 4.155%, 3.655%, and 3.155%) for calculation. When $L_l^{C_1}$, $L_h^{C_1}$, $H_l^{C_1}$, and $H_h^{C_1}$ are (160, 71, 99, 126), the maximum profit is obtained (0.0115), and the cut-off rate of 0.37 is the optimum cut-off rate C_1 (Table 11).

The results show that the optimum cut-off rates are 0.80, 0.70, 0.59, and 0.37. This indicates that when the value of a borrower is 0.80, the bank offers the benchmark interest-rate markup of 4.5%; when the value is between 0.80 and 0.70, the markup is 3%; when the value is between 0.70 and 0.59, the markup is 2.5%; when the value is between 0.59 and 0.37, the markup is 2%; and when the value is less than 0.37, 1% is added to the benchmark interest rate as the interest rate (see Figure 2).

5.3 Robustness analysis

This study employs logistic regression analysis to identify significant variables and the optimum cut-off rates of each markup interval. The incorporation of insignificant variables can affect the accuracy of the π values. Therefore, after eliminating the insignificant variables, we conduct logistic regression analysis on the significant variables to calculate the π values with greater accuracy. The results in Table 16 indicate that, excluding the deposit performance variable, all variables have a significant influence on defaults under the mutual effects of the 10 significant variables.

We take the 10 significant variables as a basis to calculate the π value of the various borrowers and use the profit calculation formula mentioned above (the profit functions 1 to 4) to identify the optimum cut-off rates (Table 17). The results show that compared to the previous points or locations for cut-off rates, the locations calculated during this analysis – namely, 0.76, 0.64, 0.56, and 0.43 - are more concentrated. This indicates that when the value of a borrower is 0.76, the bank offers the benchmark interest-rate markup of 4.5%; when the value is between 0.64 and 0.56, the markup is 2.5%; when the value is between 0.56 and 0.43, the markup is 2%; and when the value is less than 0.43, 1% is added to the benchmark



Figure 2 The optimum cut-off rates

This figure shows the optimum cut-off rates of each interest-rate markup interval. The vertical axis denotes the interest-rate markups, and the horizontal axis denotes the cut-off points from 0 to 1.

Table 16

Logistic regression analysis — Robustness test

This table presents the results of logistic regression after robustness analysis. A total of 804 valid samples are analyzed, and the variables include annual income, government preferential policies, loan period, number of guarantors, occupation, deposit performance, the period of credit-granting business, and the period of active bank accounts (9 items). B is the regression coefficient, and S. D. is the standard deviation.

Variable	В	S. D.	Т	Significance
Constant	-4.700	0.792	-5.831	0.001
Annual income	-0.001	-0.001	-1.913	0.015**
Government preferential policies	-0.598	0.250	-2.777	0.005**
Loan periods	0.108	0.179	2.503	0.004**
Number of guarantors	0.299	0.213	2.198	0.013**
Occupation	0.711	0.219	3.681	0.001**
Deposit performance	0.002	0.003	0.989	0.158
The period of credit-granting business	0.098	0.049	1.854	0.051*
The period of active bank accounts	-0.102	0.058	-1.798	0.075*
The holding of credit cards	-0.215	0.127	-1.060	0.158

** denotes a 0.05 significance level; * denotes a 0.10 significance level.

Table 17The optimum cut-off rates — Robustness test

This table presents the profits of the bank within each cut-off rate after robustness analysis (the significant variables are selected). C_1 , C_2 , C_3 , and C_4 are the cut-off rates of the five interest-rate markup intervals. L_l is the number of loans that had equivalent low forecast and actual interest rates. L_h is the number of loans that had a forecast interest rate of less than the actual interest rate. H_h is the number of loans that had a forecast interest rate of less than the actual interest rate. H_h is the number of loans that had equivalent high forecast and actual interest rates. Profit is the sum of the profits of each cut-off rate. Probability is the number of loans in the various conditions divided by the total number of loans.

Cut-			<i>C</i> ₁			<i>C</i> ₂						<i>C</i> ₃					<i>C</i> ₄				
off	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit	L_l	L_h	H_l	H_h	Profit	
		(Proba	bability)			(Probability)			(Probability)					(Probability)							
0.99	210	0	246	0	0.0049	310	0	146	0	0.0139	392	0	64	0	0.0227	428	0	28	0	0.0265	
	(0.46)	(0.00)	(0.54)	(0.00)		(0.68)	(0.00)	(0.32)	(0.00)		(0.86)	(0.00)	(0.14)	(0.00)		(0.94)	(0.00)	(0.06)	(0.00)		
0.90	210	0	243	3	0.0052	310	0	143	3	0.0142	392	0	61	3	0.0231	428	0	26	2	0.0269	
	(0.46)	(0.00)	(0.53)	(0.01)		(0.68)	(0.00)	(0.31)	(0.01)		(0.86)	(0.00)	(0.13)	(0.01)		(0.94)	(0.00)	(0.05)	(0.01)		
0.80	210	0	240	6	0.0055	310	0	140	6	0.0146	392	0	58	6	0.0235	428	0	22	6	0.0277	
	(0.46)	(0.00)	(0.53)	(0.01)		(0.68)	(0.00)	(0.31)	(0.01)		(0.86)	(0.00)	(0.13)	(0.01)		(0.94)	(0.00)	(0.05)	(0.01)		
0.76	210	0	232	14	0.0063	310	0	132	14	0.0155	392	0	50	14	0.0245	424	4	18	10	0.0279	
	(0.46)	· /	(0.51)	· /		· /	(0.00)	(0.29)	(0.03)		(0.86)	(0.00)	(0.11)	(0.03)		(0.93)	(0.01)	(0.04)	(0.02)		
0.70	210	0	226	20	0.0069	310	0	126	20	0.0162	385	4	50	16	0.0240	419	9	17	11	0.0275	
	` '	(0.00)	` '	` '		. ,	(0.00)	(0.28)	(0.04)		(0.84)	(0.01)	(0.11)	(0.04)	_	(0.92)	(0.02)	(0.04)	(0.02)		
0.64	210	1	215	30	0.0078	308	2	117	29	0.0170	385	9	40	22	0.0246	409	20	16	11	0.026	
	((0.01)	(,	(,		· /	(0.00)	(0.26)	(0.06)		(0.84)	(0.02)	(0.09)	(0.05)		(0.90)	(0.04)	(0.04)	(0.02)		
0.60	208	3	207	38	0.0084	305	6	110	35	0.0173	378	15	37	26	0.0243	395	35	14	12	0.0245	
0.54	()	· /	(0.45)	(0.08)	0.01.01	· /	(0.01)	(0.24)	(0.08)	0.0102	(0.83)	(0.03)	(0.08)	(0.06)	0.0224	(0.87)	(0.08)	(0.03)	(0.02)	0.000	
0.56	204	10	182	60	0.0101	300	16	90	50	0.0182	358	36	31	31	0.0224	361	72	11	12	0.0200	
0.50	(0.45)	()	` '	(0.13)	0.0115	(0.66)	(0.04)	(0.20)	(0.10)	0.0172	(0.78)	(0.08)	(0.07)	(0.07)	0.0202	(0.79)	(0.16)	(0.02)	(0.03)	0.014	
0.50	198	19	157	82	0.0115	283	40	72	61	0.0173	341	58	25	32	0.0202	319	115	10	12	0.0144	
0.42		· /	· /	· /	0.0155	` '	(0.09)	(0.16)	(0.13)	0.0151	(0.75)	(0.13)	(0.05)	(0.07)	0.01.40	(0.70)	(0.25)	(0.02)	(0.03)	0.0114	
0.43	196	19	116	125 (0.28)	0.0155	253	70	61	72 (0.16)	0.0151	295	105	20	36	0.0149	295	140	8	13	0.0115	
0.40	(043) 185	(0.04) 67	(0.25) 75	129	0.0138	(0.56) 230	(0.15) 95	(0.13) 58	73	0.0124	(0.65) 294	(0.23) 110	(0.04)	(0.08) 36	0.0147	(0.64) 255	(0.31) 181	(0.02)	(0.03)	0.0064	
0.40					0.0158		(0.21)	(0.13)	(0.16)	0.0124	(0.65)	(0.24)	(0.3)	(0.08)	0.0147	(0.56)	(0.40)	(0.01)	(0.03)	0.000-	
0.30	150	108	50	148	0.0120	190	145	40	81	0.0084	240	166	11	39	0.0081	180	257	4	15	-0.003	
0.50	(0.33)		(0.11)		0.0120		(0.32)	(0.09)	(0.17)	0.0004	(0.53)	(0.36)	(0.02)	(0.09)	0.0001	(0.40)	(0.56)	(0.01)	(0.03)	-0.005	
0.20	94	132	70	160	0.0084	156	180	29	91	0.0056	155	250	(0.02)	44	-0.0020	145	292	3	16	-0.007	
0.20					0.0001		(0.39)	(0.06)	(0.20)	0.0000	(0.34)	(0.55)	(0.01)	(0.10)	0.0020	(0.32)	(0.64)	(0.01)	(0.03)	0.007	
0.10	63	158	58	177	0.0070	88	259	10	99	-0.0017	124	280	6	46	-0.0056	93	345	1	17	-0.014	
	(0.14)		(0.13)				(0.57)	(0.02)	(0.22)		(0.27)	(0.62)	(0.01)	(0.10)		(0.20)	(0.76)	(0.00)	(0.04)		
0.01	31	184	32	209	0.0071	45	289	7	115	-0.0045	52	350	0	54	-0.0136	42	395	0	19	-0.020	
	(0.07)		(0.07)					(0.01)	(0.25)		(0.11)	(0.77)	(0.00)			(0.09)	(0.87)	(0.00)	(0.04)		

interest rate as the interest rate (see Figure 3). After eliminating the insignificant variables, the forecast π values are more accurate. Therefore, the identified cut-off rates are more accurate than those of the previous and the profits are greater than those of the previous calculation.

6. Conclusions

Based on the principle of profit maximization, we develop a model for optimizing the interest-rate markups of bank loans. We take 804 personal loan cases sourced from a Taiwan bank as the research samples and analyze the practical credit evaluation items used by banks for personal loans or credit-granting to identify the significant variables influencing personal credit-granting quality. Furthermore, we identify the rational interest-rate markups of loans using the model developed by the linear formula, which can provide a reference for bank loan officers to develop credit-granting strategies



Figure 3 Robustness analysis – The optimum cut-off rates

This figure shows the optimum cut-off rates of each interest-rate markup interval for the robustness analysis. The vertical axis denotes the interest-rate markups, and the horizontal axis denotes the cut-off points from 0 to 1.

and maximize bank profits.

The purpose of this study is to develop a model that can be a reference for banks. On the premise of competition, we hope to find the optimal interest rate on loans not only to avoid a rate that is too low and affects profits, but one that is too high and drives potential customers to other competitors. Doing analysis and screening of regular customers to find the optimal interest rate, of course we see that the better the conditions of the customer are, the lower the interest rate offered. However, we need to develop a model to discover the most optimal one.

For this study we reference the extent literature and the variables used in existing loan evaluation mechanisms of banks to select empirical variables. The significant variables are selected based on the empirical results and include government preferential policies, interest rates, loan periods, the number of guarantors, occupation, deposit performance, the period of credit-granting business, the period of active bank accounts, and the holding of credit cards. Regarding rationality of the interest-rate markups, under the premise and goal of profit maximization, we employ the sample information and the model developed from the linear formula to calculate the optimum cut-off rates of each markup interval. The higher the cut-off rate is, the higher the markup.

The empirical results of this study have two primary contributions to the literature. First, the results can reduce the default risk of credit-granted customers and reduce bank default rates. Second, this unprecedented study has explored the rationality of interest-rate markups. By targeting profit maximization, we utilize the model developed from the linear formula to calculate the optimum cut-off rates to determine the interest-rate markups of loans.

The management implications of this study are to develop applicable models to provide banks with the most objective and accurate method for evaluating the optimal interest rate of their loans, so as to not lose loyal customers due to a high interest rate nor lose profits due to a low interest rate. This method can serve as a calculation tool for the credit-granting unit of banks to calculate rational interest-rate markups and loan amounts. Additionally, banks can use the profit calculation formula developed herein as well as their policy needs to establish markup standards to improve the accuracy of the markup amounts for loans. This will enable the credit-granting unit of banks to offer more accurate and rational interest-rate markups for loans. Furthermore, borrowers can use this empirical model to examine the rationality of the interest rates offered by various banks when applying for loans.

Although we have conducted this study as rigorous as possible, there are still some limitations. The samples were difficult to find, and so the number of cases was not really adequate. Building up another out-of-sample case to verify our model has some difficulties. The clients included in the study were also selected and reviewed by the credit-granting unit of banks, which may also have an effect on the accuracy.

Because of the differences of loan business in several types of banks (such as public banks, SME banks, commercial banks, global banks), we recommend researchers who are interested in this topic to conduct follow-up analyses. One can compare different behaviors when speaking of a raise in the interest rate between banks, expand the numbers of samples to reduce variation, pick cases that reflect actual situations better, and use the non-performing loan ratio in order to make the study more practical.

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